Logic: A Brief Introduction
Ronald L. Hall, Stetson University

PART III - Symbolic Logic

Chapter 7 - Sentential Propositions

7.1 Introduction
What has been made abundantly clear in the previous discussion of categorical logic is that there are often trying challenges and sometimes even insurmountable difficulties in translating arguments in ordinary language into standard-form categorical syllogisms. Yet in order to apply the techniques of assessment available in categorical logic, such translations are absolutely necessary. These challenges and difficulties of translation must certainly be reckoned as limitations of this system of logical analysis.

At the same time, the development of categorical logic made great advances in the study of argumentation insofar as it brought with it a realization that good (valid) reasoning is a matter of form and not content. This reduction of good (valid) reasoning to formal considerations made the assessment of arguments much more manageable than it was when each and every particular argument had to be assessed. Indeed, in categorical logic this reduction to form yielded, from all of the thousands of possible syllogisms, just fifteen valid syllogistic forms.

Taking its cue from this advancement, modern symbolic logic tried to accomplish such a reduction without being hampered with the strict requirements of categorical form. To do this, modern logicians developed what is called sentential logic.

As with categorical logic, modern sentential logic reduces the kinds of propositions to a small and thus manageable number. In fact, on the sentential reduction, there are only two basic propositional forms: simple propositions and compound propositions.

But it would be too easy to think that this is all there is to it. What complicates the sentential formal reduction is that there are a number of different kinds of compound propositions. These different kinds of compound propositions are called propositional forms and are determined by the several different ways that simple propositions can be combined to form compound propositions. Fortunately, there is a small and manageable number of these connections, hence a relatively small number of different compound propositional forms. In this Chapter we will investigate five compound propositional forms. These forms are as follows:
1. Conjunctions
2. Negations
3. Disjunctions
4. Conditionals
5. Biconditionals.

This reduction of every compound proposition to one of these forms is almost as neat as the categorical reduction to the four standard-form categorical propositions, A, E, I, and O. However, the flexibility and power of the sentential system of logic, as we will see, is much greater than we had in categorical logic. For one thing, in sentential logic there is no necessity for reformulating arguments as standard-form syllogisms.

There is no better way to see the flexibility and power of the sentential system of symbolic logic than to jump right into an explanation of these five kinds of compound propositional forms.

Before we do this, however, we need to say something more about the basic distinction between simple and compound sentential propositions. A good place to start is with some definitions. So we will define the two as follows:

**Simple Proposition:**

An assertion (either true or false) that cannot be broken down into a simpler assertion.

Example: “The cat is on the mat.”

**Compound Proposition:**

An assertion of a combination of simple propositions.

Example: “The cat is on the mat and the dog is in the yard.”

Compound propositions are like simple propositions insofar as they are either true or false. However, the truth or falsity of a compound proposition is a function of the truth or falsity of its component simple propositions. Because this is so, sentential logic is sometimes called truth-functional logic. All this means is that the truth or falsity of a compound proposition is determined by the truth-values of its component simple propositions. We will say more as to how this determination is made as we proceed.

Sentential logic has adopted the convention of using upper case letters to stand for particular simple propositions. So in this system, C can be used to stand for the simple proposition, “The cat is on the mat.” It is common practice to use a letter that reminds us of the content of the particular proposition that we are symbolizing, but we are free to use any letter we choose. We use “C” because it reminds us of “cat.”

To symbolize a compound proposition, we simply join more than one simple proposition together, symbolizing each simple proposition with an upper case letter. So, to symbolize the compound
proposition, “The cat is on the mat and the dog is in the yard” we simply substitute the following symbolic expression: “C and D.”

The “and” here is called a truth-functional connective. It is called a connective since it connects two or more simple propositions to form the compound proposition. It is said to be a truth-functional connective because the truth-value of the compound proposition that it forms is a function of the truth-values of its component simple propositions.

One way to characterize the 5 kinds of compound propositional forms that we are now going to look at is to say that they represent 5 different truth-functional connectives, that is, 5 different ways to combine simple propositions into compound ones. The name of each of these several kinds of compound propositional forms is determined by the kind of the truth-functional connective that forms it.

Enough said then by way of introduction. Let’s turn to our first kind of truth-functional connective and accordingly to the first kind of propositional form, namely, the conjunction.

7.2 The Conjunction

Usually conjunctions are formed by joining two or more simple propositions together with the word “and.” This word “and” has a function and that function is to join two or more simple propositions together to form a compound proposition known as a conjunction. The simple propositions that are joined together to form a conjunction are called conjuncts.

The word “and” in English is not the only word that can have this function. Other English words, as well as grammatical techniques, can be surrogates for “and” and serve to function exactly as it does. Such surrogates include “but,” “nevertheless,” “moreover,” “however,” “yet,” “also,” and others, and sometimes simply a comma or semicolon. For example, it is clearly the case that the compound proposition, “The cat is on the mat and the dog is in the yard,” has exactly the same sense, reference, and truth-value as the compound proposition, “The cat is on the mat but the dog is in the yard,” or as the proposition, “The cat is on the mat; the dog is in the yard.”

As well, sometimes “and,” does not serve this function of conjoining two or more propositions together to form a conjunction. For example, “The dog and the cat were fighting each other” need not be seen as a compound proposition; that is, it can plausibly be seen as a simple proposition. Even though we have the word “and” in this sentence, it is not necessarily functioning to join two or more simple propositions together.

To capture and express the function of “and,” we will adopt the following symbol “•” (the dot) to stand for this truth-functional connective. Using this symbol, we can express symbolically the proposition, “The cat is on the mat and the dog is in the yard” as follows: “C•D.” This expression is a conjunction with two simple conjuncts “C” and “D.” The symbol expresses the “and” function whether or not the word “and” is used, but only when that function is intended in the expression that is being symbolized.
The symbol for "and" is a truth-functional connective, insofar as the truth-value of the compound proposition it forms is a function of the truth-values of its component simple conjuncts. I will specify how we determine the truth-value of a conjunction in just a moment, but first I must introduce another distinction.

It is helpful at this point to be reminded of the distinction between actual propositions and propositional forms. In sentential logic we mark this distinction by adopting the convention of using upper case letters to stand for actual propositions and lower case letters, usually “p-z,” to stand for propositional forms. Accordingly, “C” is used to stand for an actual proposition, and “p” is used to stand for any proposition whatsoever. For example, we could use “p” to stand for “C” or for “C•D” or indeed for any simple or any compound proposition whatsoever. Adopting this convention, we now see that, the symbolic expression, “C•D” may be used to stand for the particular proposition, “The cat is on the mat and the dog is in the yard.” We also see, by contrast, that the expression, “p•q” does not stand for any actual proposition. Rather, this last expression is a propositional form and can be read as standing for any conjunction whatsoever. So clearly the proposition “p•q” can stand for the particular proposition “C•D.” As we say, “C•D” is a substitution instance of the conjunctive propositional form “p•q.” The latter propositional form, however, has an infinite number of substitution instances, namely, any conjunction whatsoever. So the latter expression is much broader and much more useful than the first.

With this distinction, we are ready to specify how the truth-value of the conjunction is determined. As I have said, a conjunction is truth-functional, that is, the truth-value of the conjunction (as a compound proposition) is a function of the truth-values of its component conjuncts (simple propositions). We must also rely here on our intuitive sense for what makes a given compound proposition true or false. Clearly we would all agree that the proposition, “The cat is on the mat and the dog is in the yard” would be false if either the cat was not on the mat or the dog was not in the yard, or both. So there we have it. All we have to do is formalize this intuition. So we do so by saying that any conjunction whatsoever; (that is, any proposition that is a substitution instance of the propositional form “p•q”) is true if and only if both conjuncts (the “p” and the “q”) are true, and is false otherwise.

So far so good. But as we have been finding out, ordinary language does not always oblige our attempts to reduce it to symbolic forms. Certainly it is obvious that many expressions of conjunctions in ordinary language are not confined to such straightforward conjunctions with just two conjuncts. For example, the following sentence is clearly a conjunction of sorts: “The cat is on the mat, the dog is in the yard, and the sheep are in the meadow.” This is a conjunction, but it has three conjuncts. So is it a substitution instance of our basic propositional form for conjunction, “p•q?” Well not as it stands. We need some clarification in logic that is like the clarification provided in ordinary language by punctuation marks. With some punctuation marks, we will be able to see that our conjunction with three conjuncts is a substitution instance of the basic propositional form of the conjunction.

The punctuation that has been adopted in sentential logic is as follows: parentheses ( ); brackets [ ]; and braces { }. By convention, we use parentheses up to three times before we turn to brackets and then to
braces. To see how this works, let’s go back to our conjunction with three conjuncts. That proposition could be symbolically expressed as follows: “C•D•S.” As stated, we cannot readily see how this can be a substitution instance of our basic form “p•q” which has only two conjuncts. However, if we punctuate the given proposition it will be clear that it is a substitution instance of our basic form. To do this, we must reduce our proposition with three conjuncts to a proposition with only two conjuncts. We can do this, in two equally legitimate ways, as follows: “(C•D)•S” or “C•(D•S).” In the first case, the “p” in our form stands for the conjunction “C•D” and the “q” stands for the simple proposition “S.” In the second punctuation, the “p” in our form stands for the simple proposition “C” and the “q” stands for the conjunction “D•S.” Both of these ways of punctuating the conjunctions are equally legitimate since the truth-value of either rendering remains unchanged.

This procedure applies to any conjunction regardless of how many conjuncts it might have. We know that we have gotten our punctuation correct when we can see that the symbol for “and” clearly divides two sides of the conjunction, the “p” side and the “q” side. When this is done, we have reduced the number of conjuncts to two. The symbol for “and” that marks this divide is called the main truth-functional connective. As we will explain momentarily, the main truth functional connective determines the kind of compound proposition we have and hence the conditions under which that proposition would be true or false.

It may be helpful to define the truth conditions of “p•q” with the following table. Certainly the table makes it clear that the only time that any conjunction whatsoever is true is when both of its conjuncts are true, and it is false otherwise.

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7.3 The Negation

The second kind of compound proposition—the negation—is formed by using a truth-functional connective, also known as “negation.” What “and” is to conjunction, “not” is to negation. Again, these connectives designate a function that these terms have, namely, the function of forming connections between and among other propositions. And like the case with “and,” there are surrogates for “not,” including “it is false that,” “it is not the case that,” and others.

We will use ~ (the curl) as our symbol for “not.” Accordingly, the propositional form of negation can be symbolized as follows: “~ p.” Here this “~ p” stands for the negation of any proposition whatsoever,
compound or simple. To negate a proposition is just to assert that that proposition is not true. However, we can also assert that it is not true that some proposition is false. To make this assertion is to assert something that is true. For example, to say "It is false that the cat is on the mat" is to assert something true if the cat is not in fact on the mat." As well, double negations cancel themselves out as follows: \( \sim \sim p \) is equivalent to \( \sim p \).

In general, the symbol for "not" functions as a negation of the truth-value of whatever proposition it ranges over. The negation of a false proposition produces a true one, the negation of a true proposition produces a false one, and double negations cancel each other out. This means that if a proposition is modified by three negation signs two of them can be eliminated without affecting the truth-value of the original as follows: \( \sim \sim \sim p \) is equivalent to \( \sim p \).

If we want to express symbolically the assertion that "The cat is not on the mat," is false, we can do so with the expression "\( \sim C \)." If \( C \) is true, then "\( \sim C \)" is false. Regardless of the truth-value of "\( C \)," the expression "\( \sim C \)" is a well-formed negation since it is a substitution instance of the propositional form for negation, namely, "\( \sim p \)."

Recall, that the "\( p \)" in "\( \sim p \)" can stand for any proposition whatsoever. As such, the symbol for negation negates, or denies, or otherwise asserts that whatever proposition "\( p \)" stands for is false.

Let’s go back to our discussion of conjunctions to see how negation works in relation to other compound propositions. Let "\( p \)” in the propositional form of negation, "\( \sim p \)” stand for a conjunction "\( C \& D \)." (Let this conjunction stand for the conjunction, "The cat is on the mat and the dog is in the yard."). If this conjunction is what "\( p \)” is standing for, then "\( \sim p \)” would be expressed as: "\( \sim (C \& D) \)." What this symbolic expression asserts is that "It is false that the cat is on the mat and the dog is in the yard."

Adding the parenthesis as punctuation here, is critical, for it makes it clear that the "not" ranges over the whole conjunction and not over each conjunct. That is, what is denied here is the conjunction. And again, this is very different from denying each of the conjuncts, which we could also express, but with a different symbolic representation. To express the claim that the cat is not on the mat and the dog is not in the yard, our symbolic representation would be as follows: "\( \sim C \& \sim D \)."

If the distinction between denying a conjunction and denying both conjuncts is not clear, try this: let \( C \) stand for the proposition “Clinton was elected” and \( D \) stand for the proposition, “Dole was elected.” Clearly, it is false that both were elected; that is, the denial of the conjunction is a true assertion. Just as clearly, it is a false assertion if we were to say that it is false that Clinton was elected and it is false that Dole was elected, since Clinton was in fact elected.

When we examine the two propositions "\( \sim (C \& D) \),” and "\( \sim C \& \sim D \),” we see that they are different kinds of compound propositions. We see this by recognizing that they have different main truth-functional connectives. The main truth-functional connective in the first proposition is the symbol for negation and
hence the first proposition is a negation. The main truth-functional connective in the second expression is the symbol for “and” making it a conjunction. This is very important to notice because the truth conditions for a conjunction are different from the truth conditions of a negation.

Perhaps now we are beginning to see just how such expressions are truth-functional and that the determination of the truth of a compound proposition depends on knowing what kind of proposition it is. To make this clear let’s take our example above, letting “C” stand for the proposition “Clinton was elected” and “D” stand for the proposition “Dole was elected.” Knowing what the truth-values of “C” and “D” are, under this interpretation we know immediately that the negation “~ (C•D)” is true. Since the truth-value of the proposition in the parenthesis is false and since this proposition is negated, we know that the entire negation is true. In this case, the symbol for “not” ranges over the conjunction inside the parenthesis. Of course we know that this conjunction is false, for we know that a conjunction is true if and only if both of its conjuncts are true, and in this case one conjunct “D” is false. So to negate a false proposition (in this case the conjunction of “C” and “D”) is to assert something that is true. Hence, “~ (C•D)” is true.

The process of finding the truth-value of the second proposition, namely, the conjunction of “~ C•~ D” is quite different. A conjunction is true if and only if both of its conjuncts are true. In this case, “C” is true (Clinton was elected) and hence “~ C” is false. And even though the other conjunct “~ D” is true because “D,” which is false (Dole was not elected), is negated, making it true, a true conjunction must have both of its conjuncts be true if it is to be true. Hence, under this interpretation of “C” and “D,” “~ C•~ D” is false, and hence the two propositions “~ (C•D)” and “~ C•~ D” do not make the same assertion. “Not both” is not the same assertion as “both are not.” We must be careful not to confuse these expressions.

7.4 The Disjunction

The third kind of compound propositional form we will consider is the disjunction. Propositions of this form use “either/or” (or a surrogate expression, like “unless”) as their truth-functional connective. For example, the following two propositions are instances of this form: “Either the cat is on the mat or the dog is in the yard,” and “The cat is on the mat, unless the dog is in the yard.”

We will use “∨” (the wedge) as our symbol for the truth-functional connective “either/or.” Accordingly, the either/or proposition, “Either the cat is on the mat or the dog is in the yard” is symbolized as follows: “CvD.” The propositional form is expressed as “p∨q” and stands for any disjunction whatsoever, including of course, “CvD.” The two parts, the “either” part and the “or” part, of any disjunction (the “p” and the “q”) are called disjuncts.

When we are trying to translate sentences in ordinary language into symbolic sentential form, we will have to use the proper punctuation. For example, suppose I am trying to represent the following English sentence symbolically: “Either Clinton and Gore won the election or both did not win.” Clearly this compound proposition has an either/or structure. However, it also seems to involve two conjunctions and negations. The tokens of the conjunctions here are “and” and “both;” the token of the negations is
"not." On careful inspection, we notice that the "and" occurs in the "either" disjunct and that the "both" occurs in the "or" disjunct. We also notice the token of the negations occurs in the "or" disjunct. This leads us to see that this proposition is a disjunction of two conjunctions. We express it as follows: "(C•G) ⊨ (∼C•∼G)."

So we have a properly translated disjunction. In other words, our expression is a substitution instance of the disjunctive form "p•q."

Clearly, the main truth-functional connective in this proposition is the symbol for disjunction and this is sufficient to determine that this compound proposition is a disjunction and not, say, a conjunction, or a negation. In this disjunction, the left disjunct is a conjunction of two simple propositions and the right disjunct is a conjunction of two negated simple propositions. (Make sure you do not think of the right disjunct here as a negation. It is not. But it does involve two negations, namely, "∼C" and "∼D.")

Now we must ask what the truth conditions are in the disjunctive propositional form. Remember this compound proposition, like all of those we will consider here, is truth-functional. That is, the truth-value of the compound disjunctive assertion is a function of the truth-values of its component disjuncts. Again, we can rely on intuition to help us, at least part of the way, in making this determination. Clearly, if both disjuncts in a disjunction are false, the entire disjunction is false. And it seems just as clearly the case that if either disjunct in a disjunction is true the compound assertion is also true. We are not so sure what to say, however, about the last possibility here, namely, we are not so sure what to say about the truth-value of a disjunction both of whose disjuncts are true.

It is natural that we should wonder about this, since there is an ambiguity in the way that we use "either/or" sentences in ordinary language. Suppose that someone says, "I am either going to law school or to medical school." Let's also suppose that the person goes to law school, drops out, and goes to medical school. Do we want to say that what he or she said originally turned out to be false? It seems not. On the other hand, suppose someone says, "I am either going to marry or remain a bachelor." Now it looks like this person cannot do both. And in many other ordinary circumstances, when one says, "I will go to the movies or to the races," he or she usually means one or the other but not both. These two senses of disjunction are called its "inclusive" and its "exclusive" senses. In the inclusive sense, an "either/or" proposition is true if both of its disjuncts are true, and in the exclusive sense, an "either/or" proposition is false if both of its disjunctions are true.

So are we going to interpret disjunctions as inclusive or as exclusive? Can we do both? Or must we do one or the other? Well the fact is we need to acknowledge both senses of "either/or" expressions. But we can do this by taking the inclusive sense of disjunction as basic. This is because the exclusive sense of "either/or" is actually best expressed as a conjunction. What the exclusive disjunction asserts is something like this: "p or q and not both p and q." Or more symbolically, the exclusive disjunction can be expressed as the following conjunction:
If the sense of the sentence that we are translating seems to require the exclusive sense of disjunction we can express this with a proposition in the form of this conjunction. Otherwise, we simply take “either/or” sentences to find their most basic translation as inclusive disjunctions.

What this means is that the truth conditions for a disjunction are as follows: A disjunction is true if and only if either or both of its disjuncts are true and false if and only if both disjuncts are false. We can depict these truth-conditions as follows:

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Before we move on, we need to consider one other issue, what we might call the negative cousins of “either/or” propositions, that is, “neither/nor” propositions. These expressions can be symbolized in the following form: “~ (p • q).” If one says: I am neither going to law school nor to medical school,” the sense of this assertion is that this person is not going to law school and this person is not going to medical school. As such this “neither/nor” expression is logically equivalent to the conjunction of two negated simple propositions. In other words, this assertion about law school and medical school can be symbolized as follows: “~ L • ~ M.” But this is logically equivalent to the expression “~ (L • M)” since the only condition in which this assertion “~ (L • M)” could be true is for “L • M” to be false; and the only way that “L • M” can be false is when “L” is false and “M” is false. In summary, then, “neither/nor” expressions can be expressed as “~ (p • q),” or as “~ p • ~ q.” We must let the sense of the English sentence we are translating dictate which translation is appropriate.

7.5 The Conditional
The fourth kind of compound propositional form we will consider is the conditional (sometimes called the hypothetical). Propositions of this form use “if/then” (or a surrogate expression, like “only if”) as their truth-functional connective. For example, the following two propositions are instances of this form: “If the cat is on the mat, then the dog is in the yard,” and “The cat is on the mat, only if the dog is in the yard.”

We will use "⊃" (the horseshoe) as our symbol for the truth-functional connective “if/then.” Accordingly, the “if/then” proposition, “If the cat is on the mat, then the dog is in the yard” is
symbolized as follows: “C \rightarrow D.” The propositional form is expressed as “p \rightarrow q” and stands for any conditional proposition whatsoever, including of course, “C \rightarrow D.” The two parts of any conditional proposition have names: the “if” part is called the **antecedent** and the “then” part is called the **consequent**.

When we are trying to translate sentences in ordinary language into symbolic sentential form, we will have to use the proper punctuation. For example, suppose I am trying to represent the following English sentence symbolically: “If Bush and Cheney won the election, then Gore and Lieberman did not win the election.” Clearly this compound proposition has an “if/then” structure. However, it also seems to involve a conjunction and negations. On careful inspection, we notice that the “and” occurs in both the antecedent and in the consequent of this conditional proposition. We also notice that there are two negations in the consequent. This leads us to see that this proposition is a conditional whose antecedent is a conjunction of two simple propositions and whose consequent is a conjunction of two negated simple propositions. We express it as follows: \((B \cdot C) \rightarrow (\neg G \cdot \neg L)\).

So we have a properly expressed conditional proposition, since it is a substitution instance of the conditional form “p \rightarrow q.”

Clearly, the main truth-functional connective is the symbol for “If/then” and this is sufficient to determine that this compound proposition is a conditional proposition and not, say, a conjunction, or a negation. In this conditional proposition, the antecedent is a conjunction of two simple propositions and the consequent is a conjunction of two negated simple propositions. (Again, make sure you do not think of the consequent as a negation. It is not. But it does involve two negations, namely, “\(\neg G\)” and “\(\neg L\).”)

One important way to characterize the relation between the antecedent and the consequent of a conditional proposition is to say that the antecedent represents the sufficient condition in the conditional relation and the consequent represents the necessary condition in that relation. For example, we know that oxygen is the necessary condition for the presence of fire, and that fire is a sufficient condition for the presence of oxygen. We can express this relation symbolically in the following conditional proposition: “F \rightarrow O.” This proposition asserts that “O” is the necessary condition for “F” and that “F” is the sufficient condition for “O.”

One more helpful reminder: When “if” occurs in a sentence, it is a good policy to take what it ranges over as the sufficient condition in a conditional relation. And when we find “only if” or “only” it is a good policy to take what these words range over to be the necessary condition in a conditional relation. Consider the following example: “Only women are allowed.” Here being a woman is the necessary condition for being allowed. So we would symbolize this as, “A \rightarrow W.” Now consider the difference between the following two propositions: “You can go the fair only if you do your homework,” and “You can go to the fair if you do your homework.” In the first case, doing homework is set as the necessary condition for going to the fair; in the second, doing homework is set as the sufficient condition for going
Now we must ask what the truth conditions are in the conditional propositional form. Remember this compound propositional form, like all of those we will consider here, is truth-functional. That is, the truth-value of the compound conditional propositional form is a function of the truth-values of its component parts, its antecedent and its consequent. Again, we can rely on intuition to help us, but only to some degree; indeed, we might come to some conclusions that seem to be counter-intuitive. The reason for this is that there are so many senses of the “if/then” proposition as it is ordinarily used.

Consider the following example of an ordinary use of an “if/then” assertion. “If this figure is a triangle, then this figure has only three sides.” This is what we might call an analytic or a definitional sense of “if/then.” We could symbolize this as follows: “T \implies S.” (T ≡ “this figure is a triangle”; S ≡ “this figure has only three sides.”) Clearly if “T” is true it is logically impossible for “S” to be false. As such, this proposition asserts a relation between “T” and “S” that is stronger than other possible uses of “if/then” assertions. In fact, this relation is an “if and only if” relation. What this means is that it is logically impossible for “T” and “S” to have different truth-values.

Surely we sometimes use conditional propositions in such a way that it is logically possible for the antecedent and the consequent to have different truth-values and nevertheless be true. Consider this proposition: “If I go to the movies, then I will see Jane” (“M \implies J”). Suppose that the antecedent of this proposition is false, would the consequent also have to be false in order for this conditional assertion to be true? Surely not. Indeed, if “J” is true (I do see Jane), and if “M” is false (I do not go to the movies) the conditional proposition could still be true.

As well, we sometimes use “if/then” assertions in a “causal” sense. For example, the following proposition asserts such a causal relation: “If I turn the light switch to the “on” position, then the lights will immediately come on.” We might symbolize this proposition as follows: “T \implies O.” The relation between “T” (turning the switch to the “on” position) and “O” (the lights immediately coming on) is not a logical relation but a physical one. Suppose that as a matter of fact I turn the switch to the “on” position and it just so happens that the lights do immediately come on, that is, suppose that both “T” and “O” are true, is the conditional proposition “T \implies O” true? Well, if it so happens that there is no physical connection between the switch and the lights (they are controlled actually by a photoelectric sensor), we would likely say that the conditional is false. In this case the antecedent and the consequent are true but the conditional proposition is thought to be false.

And there are other senses of “if/then” in our ordinary usages. So which of these is the most basic? If the conditional proposition is to be treated as a truth-functional proposition, we must say how the truth of the entire “if/then” proposition is a function of the truth of its components, that is, its antecedent and its consequent. As it happens, logicians have agreed what a conditional proposition asserts most basically is logically equivalent to the following claim: “either not-p or q.” More formally expressed, “p \implies q” is logically equivalent to “\neg p \lor q.” This reading of the “if/then” does seem to capture a basic sense
of such an ordinary assertion as, “If I go to the movies then I will see Jane” since this would be equivalent to saying, “Either I do not go to the movies or I will see Jane.”

To interpret an “if/then” proposition in this basic sense is to interpret it as asserting what logicians call a conditional relation of material implication.

Since we have already established the truth conditions for the disjunction and negation, and since we are adopting as basic the sense of an “if/then” proposition as what is captured in the logically equivalent expression “¬p ⊃ q,” we can now establish the truth-conditions for the conditional. The following table should show this clearly:

<table>
<thead>
<tr>
<th>p</th>
<th>q</th>
<th>¬p∨q</th>
<th>p ⊃ q</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>T</td>
<td>T</td>
</tr>
</tbody>
</table>

In summary, what this means is that the truth conditions for a conditional are as follows: a conditional proposition is false if and only if its antecedent is true and its consequent is false, and it is true in every other case.

### 7.6 The Biconditional

The fifth kind of compound propositional form we will consider is the biconditional. Propositions of this form use “if and only if” (or a surrogate expression, like “necessary and sufficient”) as their truth-functional connective. For example, the following two propositions are instances of this form: “The cat is on the mat if and only if the dog is in the yard,” and “The cat is on the mat is the necessary and sufficient condition for the dog’s being in the yard.”

We will use “≡” (the triple bar) as our symbol for the truth-functional connective “if and only if.” Accordingly, the “if and only if” proposition, “The cat is on the mat if and only if the dog is in the yard” is symbolized as follows: “C≡D.” The propositional form is expressed as “p≡q” and stands for any conditional proposition whatsoever, including of course, “C≡D.”

When we are trying to translate sentences in ordinary language into symbolic sentential form, we will have to use the proper punctuation. For example, suppose I am trying to represent the following English sentence symbolically: “Bush and Cheney were elected if and only if Gore and Lieberman were not
elected.” Clearly this compound proposition has an “if and only if” structure. However, it also involves conjunctions and negations. To make it clear that the biconditional is the main truth-functional connective, we express it as follows: “(B•C) • (~G•~ L).”

So we have a properly expressed conditional proposition (it is a substitution instance of the conditional form “p≡q”).

Now we must ask what the truth conditions are in the biconditional propositional form. Remember this compound proposition, like all of those we will consider here, is truth-functional. That is, the truth-value of the compound biconditional assertion is a function of the truth-values of its component elements. All we have to do to establish the truth conditions of the biconditional is to recognize that its sense is captured by the following conjunction: “(p⊃q) • (q⊃p).” What this conjunction asserts is that “p” is the necessary and sufficient condition for “q” and that “q” is the necessary and sufficient condition for “p.” To interpret an “if and only if” proposition in this sense is to interpret it as asserting what logicians call a relation of material equivalence.

We can now say that the truth-conditions for the biconditional form “p≡q” are identical to truth conditions of the conjunction of two corresponding conditional propositions, “(p ⊃ q)” and “(q ⊃ p).”

The following table should show this clearly:

<table>
<thead>
<tr>
<th>p</th>
<th>q</th>
<th>(p⊃q) • (q⊃p)</th>
<th>p≡q</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
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In summary, what this means is that the truth conditions for a biconditional are as follows: A biconditional proposition is true if and only if both sides of it have the same truth-value, and it is false otherwise.
Kinds of Sentential Propositions:

**The Conjunction**: Read: “p and q.” The conjunction is symbolized as $p \& q$. The parts of the conjunction are called conjuncts. The conjunction is true if and only if both of its conjuncts are true.

**The Disjunction**: Read: “either p or q.” The disjunction is symbolized as: $p \lor q$. The parts of the disjunction are called disjuncts. The disjunction is true if and only if either or both of its disjuncts is (are) true.

**The Negation**: Read: “not p.” The negation is symbolized as: $\sim p$. The negation is true if and only if what it negates is false, and false if and only if what it negates is true.

**The Conditional**: Read: “if p then q.” The conditional is symbolized as: $p \supset q$. It has two parts: the “if” part is the antecedent and the “then” part is the consequent. The conditional is false only if the antecedent is true and the consequent is false.

**The Biconditional**: Read: “p if and only if q.” The biconditional is symbolized as: $p \equiv q$. The biconditional is true if and only if p and q have the same truth-value and false if and only if p and q have different truth-values.