Chapter 9- Sentential Proofs

9.1 Introduction

So far, we have introduced three ways of assessing the validity of truth-functional arguments. The methods for proving validity are as follows:

1. **The Truth Table Method:** We can prove that a particular argument is valid if the complete interpretation of the conditional sentence that represents that argument is shown by a truth table to be tautological.
2. **The Short-Cut Method of Assigning Truth-Values:** We can prove that a particular argument is valid if it is impossible for the antecedent of the conditional sentence that expresses it to be true when the consequent of that sentence is assigned the truth-value of false.
3. **The Valid Argument Form Method:** We can show that a particular argument is valid if it is a substitution instance of one of the four valid argument forms we have introduced so far (Modus Ponens, Modus Tollens, Hypothetical Syllogism, Disjunctive Syllogism).

These same three methods can be used for **proving invalidity**, as follows:

1. **The Truth Table Method:** We can prove that a particular argument is invalid if the complete interpretation of the conditional sentence that represents that argument is shown by a truth table not to be tautological.
2. **The Short-Cut Method of Assigning Truth-Values:** We can prove that a particular argument is invalid if it is possible for the antecedent of the conditional sentence that expresses it to be true when the consequent of that sentence is assigned the truth-value of false.
3. **The Invalid Argument Form Method:** We can show that a particular argument is invalid if it has one of the two invalid argument forms as its specific form. (The two invalid argument forms are: (a) The Fallacy of Affirming the Consequent and (b) The Fallacy of Denying the Antecedent).

Certainly, these methods are sufficient for assessing the validity or the invalidity of any truth-functional argument. Nevertheless, for some complex arguments these methods, especially the truth table method, can be very cumbersome. Consider the following argument:

If I go to law school then I will not go to medical school. If I do not go to medical school then my parents will be disappointed. My parents are not disappointed. If I do not go to law school then I will join the Peace Corps. Therefore, I will join the Peace Corps.

When we represent this argument in sentential symbols, we get the following expression:

$$L \supset \sim M$$
$$\sim M \supset D$$
$$\sim D$$
$$\sim L \supset C$$

∴ C

(L="I will go to law school" M="I will go to medical school" D="My parents will be disappointed" C = "I will join the Peace Corps"∴ = "Therefore")

Since this argument is not a substitution instance of any of the four valid forms we have studied, this method of assessing validity is ruled out. However, we can assess its validity with one of the other two methods. Assessing the validity of this argument with either the truth table method or the short-cut method is cumbersome, since in both of these methods we must translate this argument into a conditional sentence. In such arguments, this translation procedure can produce some very long and unwieldy conditional sentences. In this case, the antecedent of the conditional sentence that expresses this argument is formed by conjoining four different compound sentences. Punctuating such sentences alone is a bit of nightmare. So it seems we need another method of assessing validity that can avoid these problems.

Fortunately, we have such a method. This fourth method for determining validity is called "The Method of the Formal Proof." After some more preliminaries, we will come back to the argument above and show you how this method can demonstrate that it is valid.

This new method of proving validity will make use of the five valid argument forms that we have already studied, plus four more that we are about to introduce. However, before we introduce this new method and these four new valid argument forms, we need to go back to an important point that we have already made regarding these valid argument forms.

As we have said, valid argument forms can be expressed as rules of inference. For example, Modus Ponens is a valid argument form that can be expressed as the following rule: if an argument has one premise that is a conditional sentence and one premise that is the antecedent of that conditional sentence we can validly infer the consequent of that conditional sentence. In this chapter, we will continue to threat valid argument forms as rules of inference. As well, we will introduce five more such rules of inference: **Constructive Dilemma, Simplification, Conjunction, Addition, and Absorption**.

This list of nine rules of inference will be supplemented with a list of 10 applications of a different sort of rule. This rule is the rule of replacement (we will explain what this rule is presently). These rules then will constitute the heart of our new method of formally proving validity.

9.2 More Valid Argument Forms

1. **The rule of Simplification.** It is quite easy to see why this argument forms is valid. To see this, all we have to do is recall the truth conditions for a conjunction. Given that conjunctions are true if and only if both conjuncts are true, it should be obvious that we can validly deduce from this the truth of either conjunct.
2. **The rule of Conjunction.** By the same reasoning, if we assume that two sentences are true, then surely we can validly deduce that the conjunction of these two sentences must also be true.
3. **The rule of Addition.** Knowing that a disjunction is true if one of its disjuncts is true, then it should be obvious that if we assume that a sentence is true, then that sentence disjoined with any other sentence will also be true.
4. **The rule of Absorption.** If we know that a conditional sentence of the form "p⊃q" is true, that is, "If I go to the movies then I will see Jane" is true, then surely it follows that "If I go to the movies then I will go to the movies and see Jane" must also be true. In other words, the antecedent of a conditional sentence can be absorbed into the consequent of that sentence if it is conjoined with the original consequent. So, from "p⊃q" it is valid to infer "p⊃(p •q)."

5. **The rule of Constructive Dilemma.** If two conditional sentence are true, and the disjunction of their antecedents is true, then we can validly deduce the disjunction of their consequents.

There are some cautions I would like to issue regarding the application of two of these rules. First, it often happens that students confuse Conjunction and Addition. This confusion can easily be avoided if you remember that adding is disjoining and hence not the same as conjoining. The rule of addition says is that we can "add" any sentence to a true sentence without changing the truth-value of the original sentence. So, for example, from the sentence "P•R" we can validly deduce"(P•R) v (M⊃L)."

The application of this rule should give you no trouble if you remember that "adding" is not the same as "conjoining. That is, we add with a "wedge" (v) not a "dot" (•). Secondly, we can use the rule of Simplification only on conjunctions. In other words, this rule does not apply to any truth-functional connective other than the "dot" (•). That is, we cannot simplify disjunctions, negations, conditionals or biconditionals. In order to apply the rule of Simplification correctly, you must make sure that the main truth-functional connective of the sentence that you are simplifying is the "dot" (•).

This last point should make it clear that we cannot use the rule of Simplification on a part of a sentence. Consider the following sentence: "(P•W) ⊃(R•W). It would be an incorrect application of the rule of Simplification to deduce the conjuncts of either of the conjunctions contained in this sentence. For example, from this sentence, we cannot validly deduce "P" (or "W" or "R") by the rule of Simplification. The reason for this is that this sentence is a conditional sentence and not a conjunction. **We can only simplify conjunctions.**

With these cautions, we can now summarize all nine of the sentential rules of inference. That summary is reflected in the following list:

1. **Modus Ponens (MP) p ⊃ q; p; Therefore q**
2. **Modus Tollens (MT) p ⊃ q; ~q; Therefore ~p**
3. **Hypothetical Syllogism (HS) p ⊃ q; q ⊃ r; Therefore p ⊃ r**
4. **Constructive Dilemma (CD) (p ⊃ q) • (r ⊃ s); p v r; Therefore q v s**
5. **Disjunctive Syllogism (DS (p v q; ~q, Therefore p (p v q; ~p; Therefore, q)**
6. **Simplification (SIMP) p • q Therefore p (p • q Therefore q)**
7. **Addition (ADD) p Therefore p v q**
8. **Conjunction (CONJ) p; q Therefore p • q**
9. **Absorption (ABS) p ⊃ q, Therefore p ⊃ (p • q)**

9.3 The Method of Deduction

Every valid argument has the following form: "If P is true, then Q must be true." Put differently, every valid argument has the form of a tautological assertion of material implication. Each of our valid argument forms can be expressed in just this way. For example, the rule of Modus Ponens tells us that if the sentence "P⊃Q" is true and the sentence "P" is true, then "Q" must be true. This rule of inference can be expressed as the following tautological assertion of material implication:
"((P⊃Q) • P) ⊃**Q**."

This "if/then" structure of deductions is the centerpiece of the method of formal proofs. A formal proof consists of a sequence of "if/then" deductions that validly follow from the premises of the argument. In order for this sequence of deductions to be valid, each step in the deduction must be valid. If the conclusion of the argument can be reached by making such a series of valid deductions then the argument is proven to be valid.

Now here is where our rules of inference come into the picture. Each step that is taken in a deduction must be justified as a valid inference. If each step in the deduction is a substitution instance of a valid argument form, then this step is valid, and hence is justified.

Perhaps the best way to grasp this method of proving validity is to use an example. So let's return to the argument that was stated at the outset of this chapter. Recall that argument was as follows:

If I go to law school then I will not go to medical school.
If I do not go to medical school then my parents will be disappointed.
My parents are not disappointed.
If I do not go to law school then I will join the Peace Corps.
Therefore, I will join the Peace Corps.

This argument can be symbolically expressed as follows:

$$L \supset. \sim M$$
$$\sim M \supset D$$
$$\sim D$$
$$\sim L \supset C$$
$$\therefore C$$

In order to prove that this is a valid argument with the method of formal proof, we must show that "C" follows from the four premises of the argument. We do not have to justify the premises, since they are the assumptions of the argument, but we do need to justify every step we take in the deduction of "C." Our rules of inference will serve as our justifications for each step. So we set up the formal deduction as follows:

1)	$L \supset \sim M$	*P*
2)	$\sim M \supset D$	*P*
3)	$\sim D$	*P*
4)	$\sim L \supset C$	$P \therefore C$

This argument has four premises. "C" can be proven to follow from these premises if we can make a series of deductions from these premises (deductions that are justified by reference to our rules of inference) that end with "C" as the last line of the sequence. We put "P" to the right of each premise to indicate that this sentence is a premise of the argument. In each new line of the deduction, we must supply a justification. This justification will be a reference (using abbreviations) of one of our rules of inference.

Now let's see how we may proceed. In looking at the premises, I notice that the first two premises are substitution instances of the premises in the valid argument form known as Hypothetical Syllogism. I reason as follows: If the sentence "$L \supset \sim M$" is true, and if the sentence "$\sim M \supset D$" is also true, then Hypothetical Syllogism tells me that "$L \supset \sim M$" must be true. I indicate that I am applying the rule of Hypothetical Syllogism (HS) to steps 1 and 2 of the argument. This move is represented as follows:

1) L ⊃ ~ M P
2) ~ M⊃D P
3) ~ D P
4) ~ L⊃ C P
5) L⊃D 1, HS

We read this deduction as follows: “If 1 and 2, then 5.” Next we notice that step 3 is the denial of the consequent of the sentence we have just derived in step 5. Remembering that the rule of Modus Tollens tells us that from an assertion of material implication and the assertion of the negation of its consequent of that assertion, we can validly derive the antecedent of that assertion. So we add another step to our deduction as follows:

1) L⊃ .~ M P
2) ~ M⊃D P
3) ~ D P
4) ~ L⊃C P
5) L ⊃D 1,2, HS
\6) ~ L 3, 5 MT

We continue to read this deduction as follows: “If 5 and if 3, then 6.” Finally, we notice a relation of implication between step 4 and our newly derived step 6. Clearly, we have here an assertion of material implication (4) and an assertion of its antecedent (6). By the rule of Modus Ponens, it should be clear that “C” follows from 4 and 6. Step 7 then will be “C” and its justification will be 4, 6 MP. Our proof is now complete. We have proven that “C” follows from these premises and hence that the argument is valid. Our completed proof looks as follows:

1) L ⊃.~ M P
2) ~ M⊃.D P
3) ~ D P
4) ~ L ⊃C P
5) L ⊃D 1, 2 HS.
6) ~ L 3, 5 MT
7) C 4, 6 MP

9.4 The Replacement Rule

Even though we have already been using the notion of logical equivalence in our previous discussions, we have not yet given it a formal and precise definition. What we have gathered so far is that two sentences or sentenceal forms are logically equivalent if they have exactly the same truth-values under every possible interpretation. Recall in our discussions of categorical logic we saw, for example, that an A sentence was logically equivalent to its obversion. And in our discussions of sentential logic, we saw, for example, that P⊃ Q was logically equivalent to ~ (P • ~ Q). It is now time to formalize this notion of logical equivalence.

We will use the sign "≡" to express this notion. As I have said, the proper way to read the expression "P≡Q" is to say that it is an assertion of material equivalence. That is, "P≡Q" asserts that "P" is materially equivalent to "Q" in the way that "P⊃Q" asserts that "P" materially implies "Q." Both of these expressions, "P≡ Q" and "P⊃Q" are contingent sentences. In other words, these expressions may be false or true, depending on what values their respective component sentences have. In the assertion "P⊃Q," it is not necessarily the case that "P" materially implies "Q" unless it is impossible for the assertion "P.Q" to be false. That is, if the sentence "P⊃Q" is a tautology then we say that "P" necessarily implies "Q." The same can be said for the assertion of material equivalence. In the assertion "P ≡ Q," it is not necessarily the case that "P" is materially equivalent to "Q" unless it is impossible for the assertion "P≡Q" to be false. That is, if the sentence "P≡Q" is a tautology then we say that *"P"* is necessarily equivalent to "Q." When the assertion "P≡Q" is tautological (when both sides of the triple bar cannot have different truth-values) we say that "P" is **logically equivalent** to "Q."

With this definition in mind, we can now say that the assertion "P⊃Q" is logically equivalent to ~ (P • ~ Q) because "(P⊃Q) ≡ ~ (P • ~ Q)" is tautological. To say that this biconditional is tautological is just to say that both sides of it will always have exactly the same truth-value. If the right side is true then the left side will be true; if the left side is false then the right side will be false. When two sentences always have the same truth-value they are logically equivalent.
Generalizing from this we can now define logical equivalence as follows:

Two sentences are logically equivalent if and only if their material equivalence is tautological.

Recognizing that two sentences are logically equivalent is very useful in logical proofs. The reason for this is that this recognition enables us to make some important valid deductions simply by replacing sentences in our proof with their logical equivalents. We can make such replacements since two logically equivalent sentences assert the same thing and hence are logically exchangeable. We can formulate this license to make substitutions of logically equivalent sentences as follows:

The Rule of Replacement:
Any sentence can be replaced by a logically equivalent sentence.

It is very important to recognize that this rule of replacement can be applied to part of a sentence or to the whole. In a sentence such as "(P⊃Q) ≡ ~ (P• ~ Q)," it is a legitimate application of the rule of replacement to exchange one part of the whole sentence with an equivalent expression, or exchange the whole expression with another equivalent expression. The rule of replacement can be applied to part of a sentence or to the whole.

Suppose that we are working with the following argument that looks almost like it is a substitution instance of the valid argument form known as Constructive Dilemma:

If I marry, then I will regret it.
If I do not marry, then I will regret it.
I will marry or I will not marry. Therefore, I will regret it.

If the conclusion of this argument were "Either I will regret it or I will regret it," then the argument would clearly be valid insofar as it would be a substitution instance of Constructive Dilemma. Since it is not a substitution instance of that valid form, we must use another method to test its validity. To do this we can construct a formal proof. That proof would look like this*:*

1. *M⊃R* — *P*
2. *~ M⊃R* — *P*
3. *M v ~ M* — *P*
4. *R v R* — *3,4 CD*

By using the rule of Constructive Dilemma, we can validly derive the disjunction "R ***v*** R." However, the conclusion that we want to derive is "R." Our proof is not complete. This is where the technique of using logical equivalences can help.

What we need here is a rule that would allow us to deduce "R" from "R ***v*** R." And indeed such a deduction would be legitimate if the two sentences were logically equivalent. As it turns out there is such a rule. We call this the rule of replacement. Basically the rule allows us to replace a sentence, or part of a sentence, with a sentence that is logically equivalent to it. In this case, "R" is logically equivalent to "R ***v*** R" (This can be demonstrated by constructing a truth table to interpret the sentence "(R ***v*** R) ≡ R." The table would show that this assertion of material equivalence is tautological. To establish that this biconditional is tautological is just to show that its right side always has the same truth-value as its left side.) With this established, we can replace the left hand side of the biconditional with the right side and vice versa. This replacement procedure yields a valid deduction. This reasoning allows us then validly to deduce "R" from "R ***v*** R" by replacing the second expression with the first, since the first is logically equivalent to the second. This will not quite conclude our proof, however, since we have to cite the logical equivalence to which we are applying the rule of replacement.

There are a number of such standard logical equivalences that we can cite in justifying our deductions. This standard list of logical equivalences to which we can apply the replacement rule is as follows:

Standard Logical Equivalences

1. **De Morgan's (DM) ~ (p • q) ≡ (~p v ~q); ~ (p v q) ≡ (~p • ~q)**
2. **Commutation (COMM) (p • q) ≡ (q • p); (p v q) ≡ (q v p)**
3. **Association (ASSN) [p v (q v r)] ≡ [(p v q) v r]; [p • (q • r)] ≡ [(p • q) • r]**
4. **Distribution (DIS) [p • (q v r)] ≡ [(p • q) v (p • r)]; [p ∨ (q • r)] ≡ [(p v q) • (p ∨ r)]**
5. **Double Negation (DN) p ≡ ~~p**
6. **Transposition (TRANS) (p ⊃ q) ≡ (~ q ⊃ ~ p)**
7. **Material Implication (IMP) (p ⊃ q) ≡ (~ p v q)**
8. **Exportation (EXP) [(p • q) ⊃ r] ≡ [p ⊃ (q ⊃ r)]**
9. **Material Equivalence (EQ) (p ≡ q) ≡ [(p ⊃ q) • (q ⊃ p)]; (p ≡ q) ≡ [(p • q) v (~ p • ~ q)]**
10. **Tautology (TAUT) p≡ (p v p); p≡ (p •** ***p)***

Notice that the equivalence called Tautology allows us to deduce "p" from (p v p), which is exactly the justification we were just looking for. With this application of the rule of replacement, we can now complete our proof as follows
:

1) *M⊃ R* *P*
2) *~ M⊃ R* *P*
3) *M v ~ M* *P*
4) *R v R* *3,4 CD*
5) *R* *5, TAUT*

One important thing to notice about equivalence rules is that they are different from rules of inference in that they can apply to parts of a sentence. For example, suppose that we have the following line in a proof: (~ M v ~ Q) ⊃ R. The rule of equivalence known as De Morgan's Theorem (DM) tells us that the sentence "~ M v~ Q" is logically equivalent to the sentence "~ (M • Q)." This equivalence allows us validly to deduce the following sentence: "~ (M•Q) ⊃ R." (Remember, that in using equivalences, we can make deductions by replacing the right side of the biconditional with the left side and vice versa.)

In the *Exercise Book* you will be asked to construct some proofs. To prepare you for this, you will first be asked simply to supply justifications in proofs already constructed. These justification exercises look like the following.

1) [~ A⊃ B]. E P
2) ~ H P
3) H *v* ~ O P
4) [(~ A⊃ B) • E] ⊃ O P
5) ~ O ________
6) [~A⊃ B] .[(~A⊃ B) • E] ________
7) ~ [~A⊃ B) • E] ________
8) ~ [~A⊃ B] ________
9) ~ O • ~ [~A⊃ B] ________

Our task is to supply the justifications. The first move is to step 5. How did we get there? If we look at the equivalence rules, we notice that the rule of Material Equivalence (EQ) allows us to deduce step 5 from step 3. Next, we notice that Disjunctive Syllogism (DS) warrants the move from step 4 to step 6. Now you supply the rest of the justifications. When you are finished, your completed proof should look as follows:

1) [~ A. B]. E P
2) ~ H P
3) H *v* ~ O P
4) [(~ A. B) • E]. O P
5) ~ O 2.3 DS
6) [~A. B] . [(~A. B) • E] 1, ABS
7) ~ [~A. B) • E] 4,5, MT
8) ~ [~A. B] 6, MT
9) ~ O • ~ [~A. B] 5,8 CONJ

9.3 The Assumption Rules

Before we conclude this chapter and proceed on to the exercises, we need to introduce two more rules of inference. These two rules are Conditional Proof and Indirect Proof. These rules are explained as follows.

The very structure of an argument involves making assumptions and then drawing conclusions from those assumptions. Our interest as logicians is in validity, not in truth. As logicians we want to know what follows if certain assumptions are made. Whether the sentence "A v B" is true or not is not of interest to the logician. What is of interest is what follows if this sentence and the negation of one of the disjuncts were assumed to be true. Suppose that A is assumed to be false. Then if "A v B" is assumed to be true, and if "~ A" is also assumed to be true, then we can validly deduce B.

In this deductive process there is a lot of assuming as well as deriving from these assumptions. So long as the deriving is according to the rules, then we can safely say that if the premises were true, then the conclusion must be true. **We can assume anything**, and then, following the rules of inference, derive whatever we can derive. What we establish, however, is simply a conditional of the following kind: if what we assume were true, then what we derive according to the rules of inference must also be true. As we have said time and time again, a deductive argument can be expressed as a conditional (if/then) sentence.

If we understand this "if/then" structure of an argument, then we should be able to see that we are free to assume anything and see what can be derived from that assumption. For example, if we assume "A," we can derive by the rule of addition the sentence "A v M." From this, we can conclude that "A ⊃ (A v M)."

We can generalize this rule and say that at any point in a deduction we can assume any sentence that we want, and then, on the basis of what has already been assumed, we can deduce a conditional sentence with this assumption as the antecedent and the derivation from this assumption as the consequent. This rule is called **Conditional Proof**.

Perhaps the best way to see how this work is with an example. Consider the following argument

If I go the movies, then I will see Jane and Sally. If I see either Jane or Dick, then I will be happy. Therefore, if I go to the movies, then I will be happy.

The symbolization for this argument is as follows:

M ⊃ (J•S) (J v D) ⊃ H Therefore, M⊃ H

Clearly, it is difficult to see how we might prove this conclusion with our rules on inference. However, the rule of Conditional Proof allows us to assume any sentence that we want to assume. Since our conclusion is an if/then sentence, we are led to think that if we assume "M" and could derive "H," then we would have established that if "M" then "H." To do this we simply make the assumption of "M" and indicate that it is an assumption with the justification "AP" (assumed premise). There is no need t\ indicate a reference number here for it is not being derived from any previous step in the deduction. When we are ready to close this assumption line of reasoning, we end with a conditional sentence the antecedent of which is our assumption (AP) and the consequent of which is the last line of the derivations. When we close the assumption line of reasoning, we justify this by appeal to the rule of Conditional Proof (CP) and the number of the assumption and the number of the last line of the assumed line of reasoning. To see how this works, let's use CP to complete our proof.

1)	M ⊃(J•S)	P
2)	(J v D) ⊃ H	P
3)	M	AP
4)	J•S	1, 3 MP
5)	J	4, SIMP
6)	J v D	5, ADD
7)	H	2,6 MP
8)	M ⊃ H	4,7 CP

The rule of Conditional Proof can be used not only to derive the conclusion of an argument; it can also be used at any point in a deduction to derive a conditional sentence. We cannot simply assume the conclusion that we are trying to prove, but a derivation on the basis of the assumed premise may be a step we need in order to derive the conclusion we are trying to prove. Just remember that the assumed line of reasoning must be closed before the proof can be completed.

A similar technique is found in the rule of **Indirect Proof.** The reasoning in this rule is simple: if we make an assumption that leads to a contradiction then the assumption must be false. So suppose that we are trying to prove "~ A" from a given set of premises. The rule of Indirect Proof allows us to assume any sentence whatsoever. If we can derive a contradiction from this sentence, then we are entitled to conclude that the assumption from which the contradiction was derived is false. For example, if we assume "A" and can derive a contradiction from this assumption then we can conclude "~ A." Again, we must indicate that our line of reasoning is based on an assumption (AP). When we have derived a contradiction of the form (p• ~ p) from our assumption of "A" then we can conclude "~ A" by Indirect Proof (IP).

As always, an example will certainly make the application of this rule clearer. Consider the following argument:

> If I go either to the movies or to the races, then I will see Jane and Sally.
> If I see Jane then I will not see Sally. Therefore, I did not go the movies.

Our symbolization and proof of this argument is as follows:

1)	(M v R) ⊃ (J • S)	P
2)	J ⊃ ~S	P
3)	| M	AP
4)	| M v R	3. ADD
5)	| J • S	1.4 MP
6)	| S	5, SIMP
7)	| J	5. SIMP
8)	| ~S	2,7 MP
9)	| S • ~S	6, 8 CONJ
10)	~M	3,9 IP

Like the rule of Conditional Proof, the rule of Indirect Proof can be used not only to derive the conclusion of an argument; it can also be used at any point in a deduction. Again, we cannot simply assume on the basis of the assumed premise the conclusion that we are trying to prove, but such a derivation may be a step we need in order to derive the conclusion we are trying to prove. The assumed line of reasoning must be closed before the proof can be completed.

Study Guide

Chapter 9- Sentential Proofs

Rules of Inference

1. Modus Ponens (MP) p ⊃ q; p; Therefore q
2. Modus Tollens (MT) p ⊃ q; ~q; Therefore ~p
3. Hypothetical Syllogism (HS) p ⊃ q; q ⊃ r; Therefore p ⊃ r
4. Constructive Dilemma (CD) (p ⊃ q) • (r ⊃ s); p v r; Therefore q v s
5. Disjunctive Syllogism (DS (p v q; ~q, Therefore p (p v q; ~p; Therefore, q)
6. Simplification (SIMP) p • q Therefore p (p • q Therefore q)
7. Addition (ADD) p Therefore p v q
8. Conjunction (CONJ) p; q Therefore p • q
9. Absorption (ABS) p ⊃ q, Therefore p ⊃ (p • q)

Standard Logical Equivalences

1. De Morgan's (DM) ~ (p • q) ≡ (~p v ~q); ~ (p v q) ≡ (~p • ~q)
2. Commutation (COMM) (p • q) ≡ (q • p); (p v q) ≡ (q v p)
3. Association (ASSN) [p v (q v r)] ≡ [(p v q) v r]; [p • (q • r)] ≡ [(p • q) • r]
4. Distribution (DIST) [p • (q v r)] ≡ [(p • q) v (p • r)]; [p v (q • r)] ≡ [(p v q) • (p v r)]
5. Double Negation (DN) p ≡ ~~p
6. Transposition (TRANS) (p ⊃ q) ≡ (~ q ⊃ ~ p)
7. Material Implication (IMP) (p ⊃ q) ≡ (~ p v q)
8. Exportation (EXP [(p • q) ⊃ r] ≡ [p ⊃ (q ⊃ r)]
9. Material Equivalence (EQ) (p ≡ q) ≡ [(p ⊃ q) • (q ⊃ p)]; [(p • q) v (~ p • ~ q)]
10. Tautology (Taut) p≡ (p v p); p≡ (p • p)
    - **Assumption Rules:** You may assume any proposition, at any time, in any deduction, as long as the assumption is discharged according to the rules IP or CP before concluding the deduction.
    - **Assumed Premise** (AP): The abbreviation for the justification when the Assumption Rule is employed
    - **Conditional Proof** (CP): Allows you to infer a conditional proposition from an assumed premise. The conditional proposition inferred must have the assumed premise as its antecedent and the immediately preceding line as its consequent. Step I: Assume a premise (AP). Step II: Make deductions from this premise. Step III: Discharge the Assumed Premise with a conditional proposition (AP ⊃ q)
    - **Indirect Proof** (IP): Allows you to infer the negation of an assumed premise only if a contradiction is derived from the assumed premise. Step I: Assume a premise (AP). Step II: Derive a contradiction (p • ~p). Step III: Discharge the Assumed Premise with a negation of the Assumed Premise (~AP**)**