Logic: A Brief Introduction
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PART III – Sentential Logic

Chapter 7 – Truth Functional Sentences

7.1 Introduction

What has been made abundantly clear in the previous discussion of categorical logic is that there are often trying challenges and sometimes even insurmountable difficulties in translating arguments in ordinary language into standard-form categorical syllogisms. Yet in order to apply the techniques of assessment available in categorical logic, such translations are absolutely necessary. These challenges and difficulties of translation must certainly be reckoned as limitations of this system of logical analysis.

At the same time, the development of categorical logic made great advances in the study of argumentation insofar as it brought with it a realization that good (valid) reasoning is a matter of form and not content. This reduction of good (valid) reasoning to formal considerations made the assessment of arguments much more manageable than it was when each and every particular argument had to be assessed. Indeed, in categorical logic this reduction to form yielded, from all of the thousands of possible syllogisms, just fifteen valid syllogistic forms.

Taking its cue from this advancement, modern symbolic logic tried to accomplish such a reduction without being hampered with the strict requirements of categorical form. To do this, modern logicians developed what is called sentential logic.

As with categorical logic, modern sentential logic reduces the kinds of sentences to a small and thus manageable number. In fact, on the sentential reduction, there are only two basic sentential forms: simple sentences and compound sentences. (Wittgenstein called simple sentences "elementary sentences.")

But it would be too easy to think that this is all there is to it. What complicates the sentential formal reduction is that there are a number of different kinds of compound sentences. These different kinds of compound sentences are called sentential forms and are determined by the several different ways that simple sentences can be combined to form compound sentences. Fortunately, there is a small and manageable number of these connections, hence a relatively small number of different compound sentential forms. In this Chapter we will investigate five compound sentential forms. These forms are as follows:

1. Conjunctions
2. Negations
3. Disjunctions
4. Conditionals
5. Biconditionals.

This reduction of every compound sentence to one of these forms is almost as neat as the categorical reduction to the four standard form categorical sentences, A, E, I, O. However, the flexibility and power of the sentential system of logic, as we will see, is much greater than we had in categorical logic. For one thing, in sentential logic there is no necessity for reformulating arguments as standard-form syllogisms.

There is no better way to see the flexibility and power of the sentential system of symbolic logic than to jump right into an explanation of these five kinds of compound sentential forms.

Before we do this, however, we need to say something more about the basic distinction between simple and compound sentential sentences. A good place to start is with some definitions. So, we will define the two as follows:

Simple Sentence: A sentence (either true or false) that cannot be broken down into a simpler sentence
Example: "The cat is on the mat."
Compound Sentence: A sentence (either true or false) consisting of a combination of simple sentences
Example: "The cat is on the mat and the dog is in the yard."

Compound sentences are like simple sentences insofar as they are either true or false. However, the truth or falsity of a compound sentence is a function of the truth or falsity of its component simple sentences. Because this is so, sentential logic is sometimes called truth functional logic. All this means is that the truth or falsity of a compound sentence is determined by the truth values of its component simple sentences. I will say more as to how this determination is made as we proceed.

Sentential logic has adopted the convention of using upper case letters to stand for particular simple sentences. So, in this system, C can be used to stand for the simple sentence, "The cat is on the mat." It is common practice to use a letter that reminds us of the content of the particular sentence that we are symbolizing, but we are free to use any letter we choose. We use "C" because it reminds us of "cat."

To symbolize a compound sentence, we simply join more than one simple sentence together, symbolizing each simple sentence with an upper-case letter. So, to symbolize the compound sentence, "The cat is on the mat and the dog is in the yard" we simply substitute the following symbolic expression: "C and D."

The "and" here is called a truth functional connective. It is called a connective since it connects two or more simple sentences to form the compound sentence. It is said to be a truth functional connective because the truth value of the compound sentence that it forms is a function of the truth values of its component simple sentences.

One way to characterize the kinds of compound sentential forms that we are now going to look at is to say that they represent different truth functional connectives, that is, different ways to combine simple sentences into compound ones. The name of each of these several kinds of compound sentential forms is determined by the kind of the truth functional connective that forms it.

Enough said then by way of introduction. Let's turn to our first kind of truth functional connective and accordingly to the first kind of sentential form, namely, the conjunction.

7.2 The Negation

Before we consider four standard kinds of compound sentences, we must consider what is called the negation. This is not a separate compound sentence but a way of negating a simple or a compound sentence. To form a negation, we use the tilde ~ (the curl) as our symbol for "not." To negate a sentence is just to claim that this sentence is not true. A negated false sentence produces a true sentence; a negated true sentence produces a false sentence.

In general, the symbol for "not" functions as a negation of the truth value of whatever sentence it ranges over. The negation of a false sentence produces a true one, the negation of a true sentence produces a false one, and double negations cancel each other out. This means that if a sentence is modified by three negation signs two of them can be eliminated without affecting the truth value of the original as follows: ~ ~ ~ C is equivalent to ~ C.

If we want to express symbolically the sentence "The cat is not on the mat," we can do this by asserting that the sentence C ("The cat is on the mat") is false. We express this with "~ C." If C is true, then "~ C" is false.

Regardless of the truth value of “C,” the expression “~ C” has the opposite truth value. You can claim that it is true the cat is on the mat by negating the sentence (“The cat is not on the mat.”).

The important thing to remember here is that the negation forms a kind of true/false sentence. A sentence is negation if and only if the ~ (the curl) ranges over the whole sentence, whether the sentence is simple or compound. This will become clearer in what follows.

7.3 The Conjunction

Conjunctions are formed by joining two or more simple sentences together with the word “and.” This word “and” has a function and that function is to join two or more simple sentences together to form a compound sentence known as a conjunction. The simple sentences that are joined together to form a conjunction are called **conjuncts**.

The word “and” in English is not the only word that can have this function. Other English words, as well as grammatical techniques, can be surrogates for “and” and serve to function exactly as it does. Such surrogates include “but,” “nevertheless,” “moreover,” “however,” “yet,” “also,” and others, and sometimes simply a comma or semicolon. For example, it is clearly the case that the compound sentence, “The cat is on the mat and the dog is in the yard,” has exactly the same sense, reference, and truth value as the compound sentence, “The cat is on the mat but the dog is in the yard,” or as the sentence, “The cat is on the mat; the dog is in the yard.”

As well, sometimes “and,” does not serve this function of conjoining two or more sentences together to form a conjunction. For example, “The dog and the cat were fighting each other” need not be seen as a compound sentence; that is, it can plausibly be seen as a simple sentence. Even though we have the word “and” in this sentence, it is not necessarily functioning to join two or more simple sentences together.
To capture and express the function of “and,” we will adopt the following symbol "•" to stand for this truth-functional connective. Using this symbol, we can express symbolically the sentence, “The cat is on the mat and the dog is in the yard” as follows: “C•D.” This expression is a conjunction with two simple conjuncts “C” and “D.” The symbol expresses the “and” function whether or not the word “and” is used, but only when that function is intended in the expression that is being symbolized.

The symbol for "and" is a truth-functional connective, insofar as the truth value of the compound sentence it forms is a function of the truth values of its component simple conjuncts. We will specify how we determine the truth value of a conjunction in just a moment, but first we must introduce another distinction.

With this distinction, we are ready to specify how the truth value of the conjunction is determined. As we have said, a conjunction is truth-functional, that is, the truth value of the conjunction (as a compound sentence) is a function of the truth values of its component conjuncts (simple sentences). We must also rely here on our intuitive sense for what makes a given compound sentence true or false. Clearly, we would all agree that the sentence, “The cat is on the mat and the dog is in the yard” would be false if either the cat was not on the mat or the dog was not in the yard, or both. So, there we have it. All we have to do is formalize this intuition. So, we do so by saying that any conjunction whatsoever is true if and only if both conjuncts are true and is false otherwise.

So far so good. But as we have been finding out, ordinary language does not always oblige our attempts to reduce it to symbolic forms. Certainly, it is obvious that many expressions of conjunctions in ordinary language are not confined to such straightforward conjunctions with just two conjuncts. For example, the following sentence is clearly a conjunction of sorts: “The cat is on the mat, the dog is in the yard, and the sheep are in the meadow.” This is a conjunction, but it has three conjuncts. So, it is not well-formed since it has more than two conjuncts. It needs punctuation. We need some clarification in logic that is like the clarification provided in ordinary language by punctuation marks. With some punctuation marks, we will be able to see that our conjunction with three conjuncts can be reduce to a well-formed conjunction with only two conjuncts.

The punctuation that has been adopted in sentential logic is as follows: parentheses ( ); brackets [ ]; and braces { }. By convention, we use parentheses up to three times before we turn to brackets and then to braces. To see how this works, let's go back to our conjunction with three conjuncts. That sentence could be symbolically expressed as follows: "C•D•S" As stated, we cannot readily see how we can apply the truth conditions which make a conjunction true. Recall, a conjunction is true if and only if both of its conjuncts are true. However, if we punctuate the given sentence, it will be clear that it has only two conjuncts, a left-hand conjunct and a right-hand conjunct. We can do this, in two equally legitimate ways, as follows: "(C•D) • S" or "C•(D•S)." In the first case, the (C•D) is the left-hand conjunct (also itself a conjunction) and the S is the right-hand conjunct. In the second case, C is the left-hand conjunct and (D•S) is the right-hand conjunct. Both of these ways of punctuating the conjunctions are equally legitimate.

This procedure applies to any conjunction regardless of how many conjuncts it might have. We know that we have gotten our punctuation correct when we can see that the symbol for "and" clearly divides two sides of the conjunction, the "right-hand side" and the "the left-hand side". When this is done, we have reduced the number of conjuncts to two. The symbol for "and" that marks this divide is called **the main truth functional connective.** As we will explain momentarily, the main truth functional connective determines the kind of compound sentence we have and hence the conditions under which that sentence would be true or false.

It may be helpful to define the truth conditions of "P•Q" with the following table. Certainly the table makes it clear that the only time that any conjunction whatsoever is true is when both of its conjuncts are true, and it is false otherwise.

P	Q	P • Q
T	T	T
T	F	F
F	T	F
F	F	F

Let's go back to our discussion of negations to see how negation works in relation to other compound sentences. A curl (~) can negate a simple or a compound sentence. Consider the conjunction, C• D. The curl can negate this conjunction. But we must be careful. "~ (C• D) is not the same as "~ C• ~ D." The curl negates what it modifies. What this symbolic expression asserts that "It is false that the cat is on the mat and the dog is in the yard." Adding the parenthesis as punctuation here, is critical, for it makes it clear that the "not" ranges over the whole conjunction and not over each conjunct. That is, what is denied here is the conjunction. And again, this is very different from denying each of the conjuncts, which we could also express, but with a different symbolic representation. To express the claim that the cat is not on the mat and the dog is not in the yard, our symbolic representation would be as follows: "~ C• ~ D."

If the distinction between denying a conjunction and denying both conjuncts is not clear, try this: let L stand for the sentence "Abraham Lincoln was elected President" and D stand for the sentence, "Stephen Douglas was elected." Clearly, it is false that both were elected; that is, the denial of the conjunction is a true sentence. Just as clearly, it is a false sentence if we were to say that it is false that Lincoln was elected and it is false that Douglas was elected, since Lincoln was in fact elected.

When we examine the two sentences "~ (L•D)," and "~ L•~ D," we see that they are different kinds of sentences. We see this by recognizing that they have different main truth functional symbols. The main truth functional symbol in the first sentence is the symbol for negation and hence the first sentence is a negation. The main truth functional symbol in the second expression is the symbol for "and" making it a conjunction. This is very important to notice because the truth conditions for a conjunction are different from the truth conditions of a negation.

The process of finding the truth value of sentence “~ L•~ D” is quite different from determining the truth-value of “~ (L•D).” A conjunction is true if and only if both of its conjuncts are true. In this case, “L” is true (Lincoln was elected) and hence “~ L" is false. And even though the other conjunct “~ D” is true because “D,” which is false (Douglas was not elected), is negated, making it true, a true conjunction must have both of its conjuncts be true if it is to be true. Hence, under this interpretation of “L” and “D,” “~ L•~ D” is false, and hence the two sentences “~ (L•D)” and “~ L• ~ D” are equivalent. “Not both” is not the same sentence as “both are not.” We must be careful not to confuse these expressions.

7.4 The Disjunction

The next kind of compound sentence we will consider is the disjunction. A disjunction is an “either/or” sentences. Although it may sound strange to you, when we connect two sentences with “unless”, the preferred translation yields a disjunction, an “either/or” sentence. For example, the best translation of “The cat is on the mat, unless the dog is in the yard” is “either C or D”.

We will use ***v*** (the wedge) as our symbol for the truth functional connective “either/or.” Accordingly, the either/or sentence, “Either the cat is on the mat or the dog is in the yard” is symbolized as follows: C ***v*** D. The two parts, the “either” part and the “or” part, of any disjunction (the “C” and the D”) are called disjuncts. Every standard form disjunction has two disjuncts.

When we are trying to translate sentences in ordinary language into symbolic sentential form, we will have to use the proper punctuation. For example, suppose I am trying to represent the following English sentence symbolically: “Either Bill and Sarah won scholarships, or both did not win scholarships.” Clearly this compound sentence has an either/or structure. However, it also seems to involve two conjunctions and negations. The tokens of the conjunctions here are “and” and “both;” the token of the negations is “not.” On careful inspection, we notice that the “and” occurs in the “either” disjunct and that the “both” occurs in the “or” disjunct. We also notice the token of the negations occurs in the “or” disjunct. This leads us to see that this sentence is a disjunction of two conjunctions. We express it as follows: “(B • S) ***v*** (~ B • ~ S).”

Clearly, the main truth functional connective in this sentence is the symbol for disjunction and this is sufficient to determine that this compound sentence is a disjunction and not, say, a conjunction, or a negation. In this disjunction, the left disjunct is a conjunction of two simple sentences and the right disjunct is a conjunction of two negated simple sentences. (Make sure you do not think of the right disjunct here as a negation. It is not. Nevertheless, it does involve two negations, namely, **“**~B” and “~ S.”)

The truth value of the compound disjunctive sentence is a function of the truth values of its component disjuncts. Again, we can rely on common usage to help us, at least part of the way, in making this determination. Clearly, if both disjuncts in a disjunction are false, the entire disjunction is false. And it seems just as clearly the case that if either disjunct in a disjunction is true the compound sentence is also true. We may not be so sure what to say, however, about the last possibility here, namely, we are not so sure what to say about the truth value of a disjunction both of whose disjuncts are true.

It is natural that we should wonder about this, since there is an ambiguity in the way that we use “either/or” sentences in ordinary language. Suppose that someone says, “I am either going to law school or to medical school.” Let’s also suppose that the person goes to law school, drops out, and goes to medical school. Do we want to say that what he or she said originally turned out to be false? It seems not. On the other hand, suppose someone says, “I am either going to marry or remain a bachelor.” Now it looks like this person cannot do both. And in many other ordinary circumstances, when one says, “I will go to the movies or to the races,” he or she usually means one or the other but not both. These two senses of disjunction are called its “**inclusive**” and its “**exclusive**” senses. In the inclusive sense, an “either/or” sentence is true if both of its disjuncts are true, and in the exclusive sense, an “either/or” sentence is false if both of its disjunctions are true.

So are we going to interpret disjunctions as inclusive or as exclusive? Can we do both? Or must we do one or the other? Well the fact is we need to acknowledge both senses of “either/or” expressions. But, we can do this by taking the inclusive sense of disjunction as basic. This is because the exclusive sense of “either/or” is actually best expressed as a conjunction. What the exclusive disjunction asserts is something like this: “P or Q and not both P and Q.” More symbolically, the exclusive disjunction can be expressed as the following conjunction: “(P v Q) • ~ (P • Q)”

If the sense of the sentence that we are translating seems to require the exclusive sense of disjunction we can express this with a sentence in the form of this conjunction. Otherwise, we simply take “either/or” sentences to find their most basic translation as inclusive disjunctions.

What this means is that the truth conditions for a disjunction are as follows: A disjunction is true if and only if either or both of its disjuncts are true and false if and only if both disjuncts are false. We can depict these truth-conditions as follows:

P	Q	P v Q
T	T	T
T	F	T
F	T	T
F	F	F

Before we move on, we need to consider one other issue, what we might call the negative cousins of “either/or" sentences, that is, **“neither/nor” sentences.** These expressions can be expressed in the following symbolic form: “~ (P ***v*** Q).” If one says: I am neither going to law school nor to medical school,” the sense of this sentence is that this person is not going to law school and this person is not going to medical school. As such this “neither/nor” expression is logically equivalent to the conjunction of two negated simple sentences. In other words, this sentence about law school and medical school can be symbolized as follows: “~ L• ~ M.” But this is logically equivalent to the expression “~ (L ***v*** M)” since the only condition in which this sentence “~ (L ***v*** M)” could be true is for “L ***v*** M” to be false; and the only way that “L ***v*** M” can be false is when “L” is false and “M” is false. In summary, then, “neither/nor” expressions can be expressed as “~ (P ***v*** Q),” or as “~ P • ~ Q.” We must let the sense of the English sentence we are translating dictate which translation is appropriate.

7.5 The Conditional

The fourth kind of compound sentential form we will consider is the conditional (sometimes called the hypothetical). Sentences of this form use “if/then” (or a surrogate expression, like “only if”) as their truth-functional connective. For example, the following two sentences are instances of this form: “If the cat is on the mat, then the dog is in the yard,” and “The cat is on the mat, only if the dog is in the yard.”

We will use "⊃" (we call this the horseshoe) as our symbol for the truth functional connective “if/then.” Accordingly, the “if/then” sentence, “If the cat is on the mat, then the dog is in the yard” is symbolized as follows: “C⊃ D.” The two parts of any conditional sentence have names: the “if” part is called the **antecedent** and the “then” part is called the **consequent**.

When we are translating sentences in ordinary language into symbolic sentential form, we will have to use the proper punctuation. For example, suppose I am trying to represent the following English sentence symbolically: “If Bill and Sarah won scholarships, then Joe and Mary did not win scholarships.” Clearly, this compound sentence has an “if/then” structure. However, it also seems to involve a conjunction and negations. On careful inspection, we notice that the “and” occurs in both the antecedent and in the consequent of this conditional sentence. We also notice that there are two negations in the consequent. This leads us to see that this sentence is a conditional whose antecedent is a conjunction of two simple sentences and whose consequent is a conjunction of two negated simple sentences. We express it as follows: “(B • S) ⊃ (~J• ~ M).”

Clearly, the **main truth functional connective** is the symbol for "If/then" and this is sufficient to determine that this compound sentence is a conditional sentence and not, say, a conjunction, or a negation. In this conditional sentence, the antecedent is a conjunction of two simple sentences and the consequent is a conjunction of two negated simple sentences. (Again, make sure you do not think of the consequent as a negation. It is not. However, it does involve two negations, namely, **“**~ J” and “~ M”).

One important way to characterize the relation between the antecedent and the consequent of a conditional sentence is to say that the antecedent represents the **sufficient condition** in the conditional relation and the consequent represents the **necessary condition** in that relation. For example, we know that oxygen is the necessary condition for the presence of fire, and that fire is a sufficient condition for the presence of oxygen. We can express this relation symbolically in the following conditional sentence: “F ⊃ O.” This sentence asserts that “O” is the necessary condition for “F” and that “F” is the sufficient condition for “O.” More generally, we say that the antecedent always expresses the sufficient condition and the consequent always expresses the necessary condition in a conditional (if/then) relationship.

One more helpful reminder: When “if” occurs in a sentence, it is a good policy to take what it ranges over as the sufficient condition in a conditional relation. In addition, when we find “only if” or “only” it is a good policy to take what these words range over to be the necessary condition in a conditional relation. Consider the following example: “Only women are allowed.” Here being a woman is the necessary condition for being allowed. Therefore, we would symbolize this as, “A ⊃W.” Now consider the difference between the following two sentences: “You can go the fair only if you do your homework,” and “You can go to the fair if you do your homework.” In the first case, doing homework is set as the necessary condition for going to the fair; in the second, doing homework is set as the sufficient condition for going to the fair. Accordingly, the first sentence is symbolized as “F ⊃ H,” and the second is symbolized as “H ⊃ F.”

Now we must ask what the truth conditions are in conditional sentences. Remember this compound sentence, like all of those we will consider here, is truth-functional. That is, the truth value of the compound conditional sentence is a function of the truth values of its component parts, its antecedent and its consequent. Again, we can rely on intuition to help us, but only to some degree; indeed, we might come to some conclusions that seem to be counter-intuitive. The reason for this is that there are so many senses of the “if/then” sentence as it is ordinarily used.

Consider the following example of an ordinary use of an “if/then” sentence. “If this figure is a triangle, then this figure has only three sides.” This is what we might call an analytic or a logical sense of “if/then.” We could symbolize this as follows: “T ⊃ S.” (T= "this figure is a triangle"; S = "this figure has only three sides.") Clearly if “T” is true, it is logically impossible for “S” to be false. As such, this sentence asserts a relation between “T” and “S” that is stronger than other possible uses of “if/then” sentences.

We sometimes use conditional sentences in such a way that it is possible for the antecedent and the consequent to have different truth values and nevertheless be true. Consider this sentence: “If I go to the movies, then I will see Jane” (“M ⊃ J”). Suppose that the antecedent of this sentence is false, would the consequent also have to be false in order for this conditional sentence to be true? Surely not. Indeed, if “J” is true (I do see Jane), and if “M” is false (I do not go to the movies) the conditional sentence could still be true.

As well, we sometimes use “if/then” sentences in a “causal” sense. For example, the following sentence asserts such a causal relation: “If I turn the light switch to the “on” position, then the lights will immediately come on.” We might symbolize this sentence as follows: “T ⊃ O.” The relation between “T” (turning the switch to the “on” position) and “O” (the lights immediately coming on) is not a logical relation but a physical one. Suppose that as a matter of fact I turn the switch to the “on” position and it just so happens that the lights do immediately come on, that is, suppose that both “T” and “O” are true, is the conditional sentence “T ⊃ O” true? Well, if it so happens that there is no physical connection between the switch and the lights (they are controlled actually by a photoelectric sensor), we would likely say that the conditional is false. In this case, the antecedent and the consequent are true but the conditional sentence is false.

And there are other senses of “if/then” in our ordinary usages. So, which of these is the most basic? If the conditional sentence is to be treated as a truth functional sentence, we must say how the truth of the entire “if/then” sentence is a function of the truth of its components, that is, its antecedent and its consequent. As it happens, logicians have agreed that what a conditional sentence asserts most basically is logically equivalent to the following two sentences: “(~ p ***v*** q) and ~ (p • ~q). This reading of the “if/then” does seem to capture a basic sense. Consider an ordinary sentence such as, “If I go to the movies then I will see Jane.” This is equivalent to saying, “Either I do not go to the movies or I will see Jane,” and “It is false that I go to the movies and do not see Jane”.

To interpret an “if/then” sentence in this basic sense is to interpret it as asserting what logicians call a conditional relation of **material implication.**

Since we have already established the truth conditions for the disjunction and negation, and since we are adopting as basic the sense of an “if/then” sentence as what is captured in the logically equivalent expression “ ~P ***v*** Q,” or ~ (P • ~Q) we can now establish the truth-conditions for the conditional. The following table should show this clearly:

P	Q	~ P ***v*** Q	~ (P • ~Q)	P⊃Q
T	T	T	T	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

In summary, what this means is that the truth conditions for a conditional are as follows: a conditional sentence is false if and only if its antecedent is true and its consequent is false, and it is true in every other case. The truth conditions for a conditional sentence will have important implications, as we will soon learn.

I also must note that the truth conditions for the conditional carries a consequent that may strike you as a bit odd. If the antecedent of a conditional opposition is false, the sentence is true regardless of the truth value of the consequent. Sentences with a false antecedent are counterfactual sentences. In these cases, no matter what the consequent is, even if obviously false, the conditional sentence is true. For example, “If I jump out of the window, I will become a butterfly” is true if it is false that I have jumped out of the window (~ J ***v*** B). Clearly if I do not jump out of the window, that is if “~ J” is true then (~ J ***v*** B) must be true. Remember that one true disjunct is sufficient to make the disjunction true, whether B is true or false. We will just have to put up with such logical consequences of reading conditional sentences as material conditionals.

7.6 The Biconditional

The next kind of compound sentential form we will consider is the biconditional. Sentences of this form use “if and only if” (or a surrogate expression, like “necessary and sufficient”) as their truth functional connective. For example, the following two sentences are biconditionals: “The cat is on the mat if and only if the dog is in the yard,” and “The cat is on the mat is the necessary and sufficient condition for the dog’s being in the yard.”

We will use "≡" (the triple bar) as our symbol for the truth functional connective “if and only if.” Accordingly, the “if and only if” sentence, “The cat is on the mat if and only if the dog is in the yard” is symbolized as follows: “C ≡ D”.

When we translate sentences in ordinary language into symbolic sentential form, we will have to use the proper punctuation. For example, suppose I am trying to represent the following English sentence symbolically: “Bill and Sarah won scholarships if and only if Joe and Mary won scholarships.” Clearly this compound sentence has an “if and only if” structure. However, it also involves conjunctions and negations. To make it clear that the biconditional is the main truth functional connective, we express it as follows:

(B • S) ≡ (~ J • ~ M)

Now we must ask: what are the truth conditions of a biconditional sentence? Remember this compound sentence, like all of those considered here, is truth-functional. That is, the truth value of the compound biconditional sentence is a function of the truth values of its component elements. All we have to do to establish the truth conditions of the biconditional is to recognize that “C ≡ D” is equivalent to either of the following two sentences:

(C ⊃ D) • (D ⊃C)
(C • D) ***v*** (~C • ~D).

To interpret an “if and only if” sentence in this sense is to interpret it as asserting what logicians call a relation of **material equivalence.**

The following table should show this clearly and establish that biconditionals are true only if they have the same truth values, and false if they have different truth values. Biconditionals are true if both sides are true and if both sides are false and biconditionals are false only if the two sides of the triple bar have different truth values.

P	Q	(P⊃Q) • (Q⊃P)	(P • Q) ***v*** (~P • ~ Q)	P ≡ Q
T	T	T	T	T
T	F	F	F	F
F	T	F	F	F
F	F	T	T	T

Study Guide

Chapter 7 – Truth Functional Sentences

The Negation: "not P." The negation is symbolized as ~ P. The negation is true if and only if what it negates is false and false if and only if what it negates is true.

The Conjunction: "P and Q." The conjunction is symbolized as P • Q. The parts of the conjunction are called conjuncts. The conjunction is true if and only if both of its conjuncts are true.

The Disjunction: "either P or Q." The disjunction is symbolized as P ***v*** Q. The parts of the disjunction are called disjuncts. The disjunction is true if and only if either or both of its disjuncts is (are) true.

The Conditional: "if P then Q." The conditional is symbolized as P ⊃ Q. It has two parts: the "if" part is the antecedent and the "then" part is the consequent. The conditional is false only if the antecedent is true and the consequent is false.

The Biconditional: "P if and only if Q." The biconditional is symbolized as P ≡ P. The biconditional is true if and only if P and Q have the same truth-value and false if and only if P and Q have different truth-values.