

## Chapter 6 - Categorical Arguments

### 6.1 Introduction

Deductive arguments sometimes take a form called a **syllogism**. A syllogism is a deductive argument that is composed of three propositions. As an argument, of course, one of those propositions is used as the conclusion of the syllogism and the other two propositions are used as the premises of the syllogism.

The first premise of a syllogism is called its **major premise**; the second premise is called the **minor premise**. The following is an example of such a syllogism:

*If I go to the movies, then I will see Jane. I did go to the movies. Therefore, I saw Jane.*

As you might recall from earlier discussions, this argument is valid because it is a substitution instance of the valid argument form known as Modus Ponens. Remember also that there is a big difference between an existential argument and an argument form. An argument form has no specific content, while existential arguments do. The existential argument above is about going to the movies and seeing Jane, but these references are logically irrelevant to its formal structure. The form of the argument could be expressed as follows:

*If p then q  
p  
Therefore q*

Given that this argument form is valid, any existential argument that is a substitution instance of it, like the one about Jane and the movies, is also valid. As you can see, being familiar with valid argument forms is a great resource for evaluating the validity of existential arguments.

### 6.2 Standard Form Categorical Arguments

In this chapter we will concentrate on a existential kind of syllogism, namely, a categorical one. We will accordingly define a **categorical syllogism** as follows:

#### **Categorical Syllogism**

A deductive argument composed of three categorical propositions, one of which serves as the conclusion of the argument and the other two of which serve as the major and minor premises respectively.

In the last chapter, we learned that categorical propositions have a Standard form. There are four such forms designated by the letters, A, E, I, and O. The following proposition is a categorical proposition, but not in Standard form: “No roses are not plants.” However, we can translate this proposition into Standard form, since what it asserts is identical to the Standard form A categorical proposition: “All roses are plants.” Had the first proposition been stated as “No roses are non-plants” it would have been in Standard form and more obviously the obverse of the A proposition.

Now we will take another step and say that the categorical syllogism also has a standard form. There are some tests to see if a categorical syllogism is in Standard form.

*1. Each of the three categorical propositions in the syllogism must be in Standard form.*

2. *The three categorical propositions must contain only three categorical terms: a major term, a minor term and a middle term.*
3. *The major term is the P-term of the conclusion and occurs only once in the major premise.*
4. *The minor term is the S-term of the conclusion and occurs only once in the minor premise.*
5. *The middle term is the class term that does not occur in the conclusion, and occurs only once in the major premise and only once in the minor premise.*
6. *The major premise is stated first, the minor premise is stated secondly, and the conclusion is stated thirdly.*

Consider the following Standard form categorical syllogism:

*Some cats are friendly.  
All cats are mammals.  
Therefore, some mammals are friendly.*

In this categorical syllogism the major term is “friendly,” the minor term is “mammals” and the middle term is “cats.” The first proposition here is the major premise, the second is the minor premise, and the third is the conclusion. As it so happens, this syllogism is valid. We will get to why it is presently.

Consider the following arguments. Can you tell why these categorical syllogisms are not in standard form?

- A. *All cats are mammals.  
All rabbits are mammals.  
Therefore, all cats are rabbits.*
- B. *Some cats are not pets.  
Some non-pets are people.  
Therefore, some cats are people.*
- C. *All animals are mammals.  
All mammals are not endangered.  
Some animals are not endangered.*

In syllogism A, the major and minor premise are in the wrong order. The major term occurs in the major premise and is always the P-term in the conclusion. In this case, the P-term of the conclusion is “rabbits.” However this term occurs in the second premise, which is the place of the minor premise, not the major premise. Syllogism A is not a standard form categorical syllogism. If we changed the order of the premises it would become standard form categorical syllogism.

In syllogism B, there are more than three class terms. In this case we have four terms, “pets,” “non-pets,” “cats,” and “people.” The premises are also in the wrong order, since the major term “people” occurs in the second premise (the minor premise) and the minor term “cats” occurs in the first premise (the major premise). As well, there is not middle term, namely a single class term that occurs in both the major and the minor premise. Syllogism B is not a Standard form categorical syllogism. In syllogism C, the minor premise is not a Standard form categorical proposition. As well, the minor and major premises are in the wrong order. Syllogism C is not a Standard form categorical syllogism.

Given these restrictions of form it should not be surprising that even though there are thousands of categorical syllogisms there is a rather small number of possible Standard form categorical syllogistic forms. We are careful to distinguish categorical syllogisms from categorical syllogistic forms. We can see this when we begin to think in terms of A, E, I, and O propositional forms. There are thousands of A propositions, but only one A form. And the same is true for the other three propositional forms, E, I, and O. Accordingly, a **syllogistic form** is partially expressed as a combination of three of these propositional forms, and would look something like, AEI, or AAA, or EIO, or some other of the possible combinations. This formal order of letters is called **the mood** of the syllogism. There are only 64 different Standard form categorical syllogistic moods.

We cannot say, however, that there are only 64 Standard form categorical syllogistic forms. The reason for this is that there are variations in the place that the middle term can occupy in a standard form categorical syllogism. These variations are called **figures**.

**The full form of a syllogism is expressed as a combination of its mood and its figure.**

There are only four standard form categorical figures. They are as follows:

**Figure #1** The middle term (M) can occur as the subject term of the major premise and the predicate term of the minor premise. This is called Figure #1. For example consider the following syllogistic form in the AAA mood. The form of a standard form categorical syllogism consists of both its mood and its figure. The following argument has the form AAA-1.

**All M are A**  
**All B are M**  
**All B are A**

**Figure #2** The middle term (M) can occur as the predicate terms of both the major and minor premises. This is called Figure #2. The following argument has the form AAA-2.

**All A are M**  
**All B are M**  
**All B are A**

**Figure #3** The middle term (M) can occur as the subject terms of both the major premise and the minor premise. This is called Figure #3. The following argument has the form AEE-3.

**All M are A**  
**No M are B**  
**No B are A**

**Figure #4** The middle term can occur as the predicate term of the major premise and the subject term of the minor premise. This is called Figure #4. The following argument has the form AOO-4.

**All A are M**  
**Some M are not B**  
**Some B are not A**

We can now calculate that with the addition of figures to the 64 possible moods, **there are 256 standard form categorical syllogistic forms**. This might sound like a lot, until we realize that there are thousands upon

thousands of existential categorical syllogisms, precisely because there are thousands upon thousands of class terms. But there is even better news. As it turns out there are **only 15 of these forms that are valid**. This is very helpful to know, for it tells us that any existential categorical syllogism that is a substitution instance of one of these valid forms will also be valid.

### 6.3 Testing Validity: The Rule Method

Fortunately we do not have to go through all of the 256 argument forms to find out which are the valid ones. Nor do we have to memorize which forms are valid. We have two methods for testing standard form categorical syllogisms to see if they are valid. The first is what we will call the **Rule Method** and the second is the method of **Venn Diagrams**. We will consider each of these methods and then do some exercises applying each method.

The rule method of testing for the validity of a standard form categorical syllogism involves, as you might imagine, knowing some basic rules that are necessarily followed in any valid categorical syllogism. If any of these rules is broken, this is sufficient for determining that the syllogism is invalid. A syllogism can break more than one rule, but it only takes breaking one to establish its invalidity. Corresponding to every broken rule is a **categorical fallacy**. If no rule is broken, this is sufficient for establishing that the syllogism is valid.

These rules and corresponding fallacies are as follows: **Valid standard form categorical syllogisms:**

(**Rule # 1**) must have only three terms, each of which designates the same class throughout.

#### **Fallacy of Illicit Terms**

(**Rule # 2**) cannot have two negative premises. **Fallacy of Two Negatives**

(**Rule # 3**) must have a negative conclusion if either premise is negative.

#### **Fallacy of Illicit Quality**

(**Rule #4**) cannot have a conclusion with a existential quantity if both premises are universal in quantity.

#### **Existential Fallacy**

(**Rule # 5**) must distribute the major term in the major premise if the major term is distributed in the conclusion.

#### **Fallacy of Illicit Major**

(**Rule # 6**) must have a distributed minor term in the minor premise if the minor term is distributed in the conclusion. **Fallacy of Illicit Minor**

(**Rule # 7**) must have a distributed middle term in at least one premise. **Fallacy of Illicit Middle**

It is easy to see how these rules can help if we apply them to the example of an invalid syllogism that we used above.

*All cats are mammals.  
No dogs are cats.  
Therefore, all dogs are mammals.*

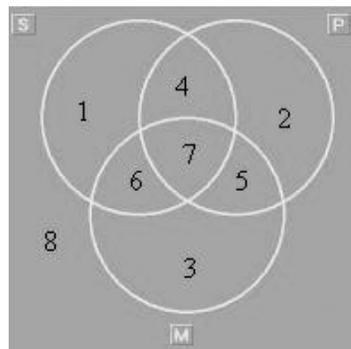
This syllogism obviously does not commit the fallacy of illicit terms, since there are only three such terms, all of which are used consistently throughout the syllogism. The fallacy of two negatives is not committed since both of the premises are not negative. The existential fallacy is not committed since the quantifier in the conclusion is not existential but universal, even though both premises are universal propositions. The fallacy of illicit major is not committed because the major term (“mammals”) is not distributed in the conclusion. The fallacy of illicit minor is not committed because even though the minor term (“dogs”) is distributed in the conclusion it is also distributed in the premise. The fallacy of illicit middle is not committed since the middle term (“cats”) is distributed in one of the premises (in fact it is distributed in both of these premises). So we come finally to the fallacy of illicit quality. Here we have an affirmative conclusion that is drawn from premises one of which is negative. Because it commits this fallacy, this syllogism is invalid.

Knowing what fallacies to avoid is helpful. And being familiar with the fallacies can make it obvious in some cases that a existential syllogism is invalid. The following exercises should help you master these rules and fallacies. In these exercises indicate which fallacy if any the argument expressed in the passage commits. The possible answers are as follows. **The Fallacy of:**

1. **Illicit Terms** (A standard form categorical syllogism can have only three terms.)
2. **Illicit Minor** (If the minor term is distributed in conclusion it must be distributed in minor premise)
3. **Illicit Major** (If the major term is distributed in conclusion it must be distributed in major premise)
4. **Illicit Middle** (Middle term must be distributed at least once)
5. **Illicit Quality** (Conclusion must be negative if there is a negative premise)
6. **Existential Fallacy** (Conclusion can't be existential if both premises are universal)
7. **Two Negatives** (Can't have two negative premises)

## 6.4 Testing Validity: Venn Diagrams

We have already used Venn Diagrams to show how the classes designated by the subject and predicate terms of a single categorical proposition are connected. Since a standard form categorical syllogism involves only three classes, the ones designated by the major term, the minor term, and the middle term, it should be obvious that we need to add a third interlocking circle to represent all three of the classes that are involved in the syllogism. Such a diagram of these three interlocking circles would have the following appearance:



Notice that there are various relations of class inclusion and class exclusion. Area 1, for example, is a section of the class of S that is excluded from both the class of P and the class of M. Area 4 is a section of S that is included in P but excluded from M. Area 7 is included in S, P, and M, and so forth.

The labels for the categorical syllogism that we want to diagram will be the letters that suggest the content of the existential argument we are testing. For example, if we are talking about pets, sheep and mammals, we would expect to use the upper case letters, P, S and M to designate our three circles respectively. But note, it does not matter which circle gets which letter. That is, in our drawing above, S could be the circle representing the middle, minor, or major class term. All we need to make sure of is that each class term is represented and labeled properly.

Remember that when we are diagramming a universal proposition, we use the technique of emptying areas of the diagram that are excluded from or included in the S-term of the proposition. In the A proposition, “All S are P,” for example, we want to show that the entire class of A is included in the class of P. We do this by emptying all of the class of A that is not in P. Our technique for doing this is to blacken that area of A that is outside of P. In the case of the universal negative proposition, “No S are P,” we want to show that the area where the class of S and P intersect is empty. Again we blacken this area to indicate this.

In the case of existential propositions, we want to indicate that the areas referred to by the S terms of these propositions are classes that are populated with at least one member (an “x”) and that “x” is either included or

excluded from the class designated by the P-term of that proposition. For example, the I proposition, “Some S are P,” asserts that there is at least one member (an “x”) in the class designated by the S term that is included in the class designated by the P-term. To indicate that the area of S that is included in P is populated, we put an “x” in that area. Similarly, if we want to show that an area of S that is outside of P is populated, (“Some S are not P.”) then we do so again by putting an “x” in that area of S that is outside of P.

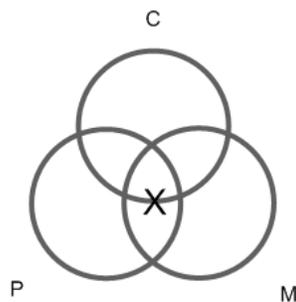
To diagram a existential proposition in a categorical syllogism, you select the common area (both parts) of the diagram as populated. Consider the following invalid syllogism:

*Some persons are mortals.*

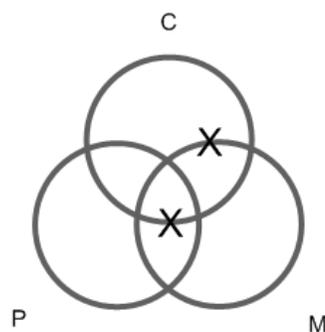
*Some creatures are not persons.*

*Therefore, some creatures are mortals.*

The common area of persons and mortals is divided into two parts by the class of creatures: things which are persons, mortals and creatures, and things which are persons, mortals and not creatures. The proposition affirms that something is a person and mortal, but not whether that something is also a creature. Since it is not clear whether the “x” is included or excluded from the class of creatures, we must place the x” on the line as follows:



Follow the same procedure in diagramming the second proposition. The second premise in this argument asserts that there exists an at least one member of the class of creatures that is excluded from the class of persons. However, no assertion is made as to whether this “x” is included or excluded from the class of mortals. Since we do not know whether the “x” is included or excluded from the class of mortals, we must put the “x” on the line that separates the class of creatures from the class of mortals. What is asserted is that something exists that is a creature and not a person: what is not asserted, however, is the claim that this “x” is or is not included in the class of mortals. To diagram the proposition you must put an “x” on the line in the area that is outside of the class of persons, but common to the class of creatures and mortals. Your diagram should look like this:



The conclusion (Some creatures are mortals) asserts that there is something which is both a creature and mortal. The diagram above **does not guarantee** that either of the parts of this common area has something (an “x”) present. In the upper common area of the diagram what is present may be only in the left hand part, and in the lower

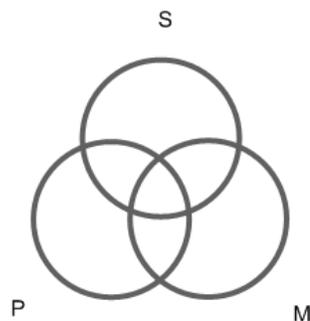
common area what is present may be only in the right hand part. Consequently, we do not know that there is something in the common area of creatures and mortals. The syllogism does not guarantee the truth of the conclusion and is therefore invalid. In determining validity, if something is not **necessarily present** in a given area, you must not assume that it is. (Hint: Any time that an “x” falls on a line, the argument diagrammed is invalid. As well, no argument with two existential premises is valid.)

We must be clear in this process that when we are testing the validity of a standard form categorical syllogism, all we do is diagram the major and the minor premises of the argument. If the diagram of the premises of an argument is sufficient also to diagram the conclusion of that argument, then the argument is valid, and invalid otherwise. Consider the following IAI-3 syllogism:

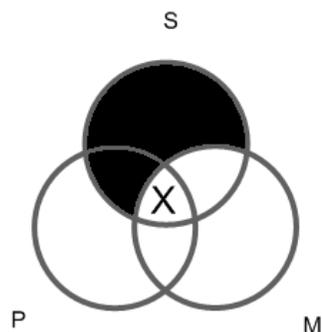
*Some sheep are pets.*  
*All sheep are mammals.*  
*Therefore, some mammals are pets.*

Is this a valid argument? Well here is what we do to find out.

**Step one:** draw three interlocking circles and label each one with an uppercase letter designating one of the classes in the argument being tested. In this case, we naturally would select (although we are free to use other selections) S, M, and P. Our drawing should look like this:



**Step two:** diagram the major and the minor premises but never diagram the conclusion. It does not matter which premise you diagram first, unless one of the premises is an existential proposition and the other one is a universal proposition. In this case, always diagram the universal proposition first. Accordingly, our diagram should have an appropriate section blackened (emptied) and an “x” should be appropriately placed as follows:



**Step three:** examine the conclusion of the syllogism and ask whether the diagram before us also diagrams the relation between the S and P-terms that is asserted by the conclusion. That is, does this diagram above show that there is an “x” in the part of the class of mammals that is included in the class of pets is populated?

Notice that all of the area of S that is not in M is blacked, that is, is empty. Now notice that the area of S that is included in the area of P is populated (shown by an “x” in that area). So again we ask, does this diagram also diagram the conclusion? Recall the conclusion is, “Some mammals are pets.” In order for this to be shown in our diagram, an area of M that is included within P must be populated, that is, there must be an “x” in the area common to M and P. And indeed this is shown by this diagram and so the argument is valid.

Now let’s consider an invalid syllogism to show you what that diagram would look like. Consider an argument of the form AEA-1. Recalling that the predicate term of the conclusion of a standard form categorical syllogism is the major term of the argument and that the subject term of the conclusion is the minor term, and that the term common to both premises is the middle term, we can begin to construct an argument of this form.

We must however, take one more thing into consideration and that is the figure, or the place of the middle term in the syllogism we are constructing. In a figure-1 syllogism, the middle term is the subject term of the major premise (always stated first) and the predicate term of the minor premise (stated second).

Formally, our syllogism will be as follows:

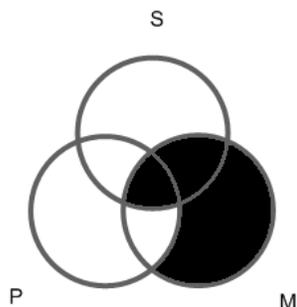
*All M are P.*  
*No S are M.*  
*All S are P.*

Now we can give our argument some content. Let's stay with **mammals**, **sheep** and **pets**.

*All mammals are pets.*  
*No sheep are mammals.*  
*Therefore, all sheep are pets.*

Does this look like a valid argument to you? It might not simply because all of the propositions in the syllogism are false. However, we must be vigilant, since arguments with false propositions can be valid. Keep in mind that validity is determined independently of questions of truth. The question of validity is simply a question of whether the conclusion follows from the premises. Certainly it is false that all sheep are pets, but the question of validity is the question of whether it would have to be true if the premises of the argument were true. And it applies in this case even though the premises in this argument are clearly false.

To settle our question of validity, let’s diagram the syllogism.



It should be obvious to you that the argument is not valid, for the diagram of the two premises is not sufficient to diagram the conclusion. Had the claim of the conclusion been shown in the diagram of the premises, the diagram would have made it clear that all of the area of S was included in the area of P. Clearly the diagram does not show this. Hence the argument diagrammed is invalid.

Our exercises will give you some practice in the method of testing syllogisms for validity by using Venn Diagrams. Before you get to these exercises, let us recall that our judgments concerning validity in these exercises do not make the existential assumption that Aristotle made. At the same time I want you to be aware of what effect making that assumption would have on the validity of the argument in question. Accordingly you will be asked to make one of three judgments concerning each argument that you diagram: the argument is, **Valid, Invalid, or Traditionally Valid**. If an argument is **valid or invalid** without qualification, it is valid or invalid even if the classes it specifies are empty. However, argument that is invalid, by virtue of the possibility that the classes it specifies may be empty, nevertheless might be valid if the existential assumption is made, that is, the assumption that every class specified in the argument has at least one member. In this case, we say that the argument is **traditionally valid**.

Obviously there are fewer argument forms that are valid without qualification than there are traditionally valid forms. The 15 argument forms that are **valid without qualification** are as follows:

**Figure 1: AAA; EAE; AII; EIO**

**Figure 2: EAE; AEE; EIO; AOO**

**Figure 3: IAI; AII; OAO; EIO**

**Figure 4: AEE; IAI; EIO**

In addition to these forms, the following are **traditionally valid**.\*

**Figure 1: AAI; EAO**

**Figure 2: AEO; EAO**

**Figure 3: AAI; EAO**

**Figure 4: AEO; EAO; AAI**

*(\*Notice that each of these traditionally valid argument forms draws a existential conclusion from universal premises. If the existential assumption is made, these inferences do not commit the existential fallacy.)*

## **6.7 Translating and Testing More Complicated Arguments**

As we noted in the last chapter, it is quite obvious that in ordinary modes of conversation, we humans do not tend to talk or write in standard form categorical arguments. Now we can see clearly that this poses a serious problem when it comes to assessing the validity of categorical syllogisms.

Again, with some reformulation efforts, most of the arguments we make in ordinary discourse can be stated so that they fit standard form categorical arguments. This reformulation enables many of the arguments that we find expressed in ordinary language to be assessed by the methods of categorical logical analysis. Let's see now how this reformulation or translation process might proceed.

Consider the following argument expressed in ordinary language:

***Every cat can be a good pet since they are all friendly, and, as we know, every friendly animal can be a good pet.***

While it may not be obvious that this argument can be expressed in standard categorical form, nevertheless it can be. Consider the following translation:

*All friendly animals are animals that can be good pets.  
All cats are friendly animals.  
All cats are animals that can be good pets.*

In order for this to be a legitimate translation, it must be logically equivalent to the argument it is a translation of. That is, the premises and the conclusion must have exactly the same content if the translation is to be legitimate. This seems to be the case in this translation. The translation seems neither to have added to, nor left out, any of the content that was expressed in the original argument.

In making this translation we find that the formal structure of the argument of the original is identical to the formal structure of the translated categorical syllogism. The two are logically equivalent expressions insofar as the original can be substituted for the translation without a loss of truth-value. As we might put this, the original argument is logically equivalent to a categorical argument of the form AAA-1. As we could easily demonstrate with a Venn diagram, this argument, as are all arguments of this form, is valid. Of course this does not imply that the argument is sound, for indeed it does have a premise that is obviously false. Are all cats friendly animals?

It is important to be able to make this translation, for now we can test it with Venn Diagrams to see if it is valid, or alternatively we could see if it is invalid by virtue of having committed any one of the seven categorical fallacies. What we hope you can readily understand here is not only that this argument is valid but that this assessment of validity applies to either of its expressions, the ordinary one or the categorical one, for the two expressions of the argument are logically equivalent.

In order to make this kind of reformulation or translation of ordinary language into standard categorical logical form, there are some helpful hints to follow, and some helpful techniques and strategies to employ. We will spend the rest of this chapter going over some of these. Our exercises at the end will give you some practice in these translation procedures.

One of the first things that we need to do is to make sure that the syllogism that we are trying to translate into standard form has only and exactly **three terms**. If the syllogism has fewer or more terms, it cannot be translated into standard form. However, in many cases there may appear to be more than four terms while actually there are only three. This happens when two different class words are employed, but in fact the two different class words are synonymous expressions and hence refer to the same class. For example, consider this argument:

*Some American citizens are not eligible to vote, and all Americans are patriotic. Hence some American patriots cannot participate in the election process.*

At first look it seems that this argument does not have the right number of classes, namely, exactly three, and so cannot be reformulated into standard categorical form. On closer inspection, however, we can determine that the class of "American citizens" is the same class as the class of "Americans." So the subject class of the first proposition (namely, the major premise, which is clearly an O proposition) seems to be identical to the subject class of the second proposition (namely, the minor premise, which is clearly an A proposition).

As well, the subject term of the conclusion, namely, "American patriots" designates exactly the same class as does the predicate term of the minor premise, namely, "patriotic," or more precisely, "those who are patriotic."

And finally, the class designated by the predicate term of the conclusion (the major term), namely, the class of those who cannot participate in the election process, can be read as identical to the class designated by the predicate term of the major premise, namely, the class of those who are not eligible to vote.

Reducing the number of terms to exactly three then, allows us to proceed with the reformulation of the argument. Indeed, we would proceed as follows. We can see that the single class of “American citizens” is the middle term. Since this term appears in the subject place of both the major and minor premise, we also see that this argument must be expressed as a standard form categorical syllogism in the third figure. After locating the major and minor terms, we recognize the mood of this syllogism as OAO. And given that it is in the third figure, the form of the translated syllogism is OAO-3.

Our reformulation, with the following three terms, A=American citizens, V=eligible (or registered) voters, and P=American patriots, yields the following standard form categorical syllogism, which by the way, could easily be shown (with a Venn diagram) to be valid:

*Some A are not V.  
All A are P.  
Hence, Some P are not V.*

In some cases we can reduce the number of terms in an argument in ordinary language by using one of our standard immediate inferences. (Recall, these inferences include: conversion, obversion and contraposition. Here is how this might work. Consider the following argument in ordinary language:

*Because many animals are not non-mammals, but no snakes are mammals, we can conclude that some animals are non- snakes.*

Our first attempt to reformulate this argument might look like this:

*Some A are not non-M.  
No S are M.  
Therefore, some A are non-S.*

Clearly, we appear to have five terms in this syllogism: A, M, non-M, S, and non-S. A little reflection will tell us, however, that by obverting the propositions that contain non-M and non-S we could change the non-M to M and the non-S to S. The first premise, “Some A are not non-M” is logically equivalent to its obverse, “Some A are M.” By the same token, the conclusion, “Some A are non-S” is logically equivalent to its obverse, “Some A are not S.” So this is what our translation with this reduction of terms to A, M and S would look like:

*Some A are M.  
No S are M.  
Therefore, some A are not S.*

Now, is this a standard form categorical syllogism? Not quite. Notice that the major term S (the predicate of the conclusion) is in the second premise (where the minor premise should go) and the minor term A (the subject of the conclusion) is in the first premise (where the major premise should go). The problem then is that the premises are not in the right order. We can fix this simply by reversing them. This would give us the following EIO-2 standard form categorical syllogism:

*No S are M.  
Some A are M.  
Therefore, some A are not S.*

As it happens, every syllogism in this mood, regardless of its figure, is valid. In other words, syllogisms of the forms EIO-1, EIO-2, EIO-3, and EIO-4, are all valid. Maybe the old song about the “Old MacDonald” will help you remember that every standard form “EIO” syllogism is valid.

Let's take one more example of a translation that involves immediate inferences. Consider the following argument:

*All nonresidents are ineligible to vote.  
No residents are college students.  
Therefore, no college students are eligible to vote.*

Again in this argument we seem to have more than three terms. In fact, it looks like we have the following five terms: nonresidents, ineligible voters, residents, college students, and eligible voters. By recognizing that the class referred to by the term “ineligible” is identical to the class referred to by the term “non-eligible” we are prepared to use one of our immediate inferences to reduce “non-eligible” to “eligible.” But first we must substitute “non-eligible” for “ineligible” in our major premise. This yields the proposition: “All non-residents are non-eligible voters.” We now notice that if we substitute the contrapositive of this reformulated first premise we are able not only to get rid of the “non” in the term “non-eligible” but we are also able to get rid of the “non” in the term “non-residents.” Accordingly, our substitution of the logically equivalent contrapositive of the first premise yields the following proposition: “All E are R” (that is, “All eligible voters are residents”). Now it is easy to put the argument into the following standard form AEE-4 (valid) syllogism:

*All E are R.  
No R are C.  
Hence, No C are E.*

One of the main things we need to do in making such translations is to be sensitive to the content of the expression that we are trying to translate into standard categorical form. Our aim always is to say exactly the same thing in categorical form that is said in the expression that we are attempting to translate. This sensitivity will depend largely on how highly developed your native skills are as a speaker when it comes to discerning the meaning of what you hear or read. Surprisingly, much agreement can be established in this artful process.

There are other special challenges for translation that we now need to address. These include: (1) The translation of arguments that contain singular propositions; (2) The translation of arguments that contain propositions in need of parameters; (3) The translation of arguments that contain conditional propositions; (4) The translation of arguments that contain exceptive propositions; and (5) the translation of sorites and enthymemes. We will take these challenges one at a time.

- **Arguments Containing Singular Propositions**

Translations that involve what is called singular propositions pose a existential challenge. Consider the following very famous deductive syllogism:

*All persons are mortal.*  
*Socrates is a person.*  
*Therefore, Socrates is mortal.*

When we try to translate the minor premise in this syllogism we are struck with the fact that “Socrates” names an individual not a class. Such propositions are called singular propositions. So how do we translate such singular propositions into assertions of class inclusion or exclusion? Clearly, this syllogism is valid, but can we show that it is with the methods of assessment available in categorical logic? Well, we cannot unless we can express this singular proposition as a standard form categorical proposition.

One reasonable suggestion is that we treat “Socrates” as a class with just one member. On this reasoning then the term “Socrates” should be taken as referring to all members of this single-member class. This would yield the following translation: “All S are H.” In turn this procedure yields the following rule: **All affirmative singular propositions should be translated as A propositions.**

The same reasoning applies to negative singular propositions. For example, “Socrates is not handsome” should be translated as referring to every member of the class of the single-member class of Socrates and asserting that none of it is included within the class of handsome persons. Accordingly, the translation of this negative singular proposition should be, “No S are H.” This procedure can be generalized to the following rule: **All negative singular propositions should be translated as E propositions.**

There is just one snag in these translation procedures. This, again, is something that did not occur to Aristotle, namely, the possibility of a class being empty. Universal propositions, affirmative or negative, do not make the existential assumption. (Recall that the existential assumption is the assumption that a class has members.) Clearly most singular propositions, affirmative or negative, make the existential assumption. (Of course some singular propositions do not make this assumption. For example: “Santa Claus is jolly” However the proposition “Socrates is a person” not only asserts that every member of this single-member class is included within the class of persons, but also asserts that this class is not empty, or positively, that it has one member. So how do we capture this existential assertion when we express singular propositions as universal propositions?

One suggestion is that we just translate singular propositions as two propositions, one universal and one existential. So the proposition “Socrates is a person” would find its proper translation as the conjunction of the following two propositions: “All S are P” and “Some S are P.”

This translation of singular propositions into two standard form categorical propositions may well be the best way to render them. Unfortunately, if we follow this procedure then the syllogism we are investigating would not be a standard form syllogism because it would have three instead of the standard two premises; more precisely, this procedure would yield two minor premises in this syllogism and hence it would not be in standard form and hence could not be assessed in terms of the methods for determining validity available in categorical logical analysis. As we might put this, arguments with singular propositions are **asyllogistic**.

In order to avoid excluding syllogisms with singular propositions from categorical analysis, logicians have simply adopted the following policy: the preferred translation of an affirmative singular proposition is to translate it as an A proposition; the preferred translation of a negative singular proposition is an E proposition.

In this course, we will do the same, with one qualification. In order to acknowledge and capture the existential assumption that may be involved in the assertion of a singular proposition, we will say that the Venn diagram of such a proposition may reflect a existential translation of the singular proposition if it turns out that the universal translation yields an invalid argument. If you are translating a singular proposition in an exercise and it

turns out to yield an invalid argument, go back and re-translate it as an existential proposition. If the Venn diagram now shows that this argument is valid, then the original argument is valid. (This is what we might call the principle of charity. On this principle the most plausible translation of an argument is the one, if any, that shows it to be valid.) If every possible rendering of the propositions continues to show that an argument is invalid, then the original argument is invalid. (Follow the same procedure for negative singular propositions.)

Now consider the following argument:

**Tyco is noted for being able to run extremely fast.  
Tyco is a cheetah and hence some cheetahs are fast runners.**

Notice that there are no explicit quantifiers in the premises of this argument. So we must supply them. The first premise is a singular proposition, and so we translate it as "All T are F" (F="fast runners"). The second premise is also a singular proposition and can be translated as "All T are C."

Next we turn to the conclusion. This is an existential proposition and can be translated as "Some C are F." Now our syllogism should look like this:

**All T are F.  
All T are C.  
Some C are F**

If you do a Venn diagram of this argument you will see that it is not valid. You may also note that it commits the existential fallacy. However, if we go back and re-translate the second premise as "Some T are C," which we are allowed to do because it is a singular proposition, such a re-translated argument would be of the following AII-3 form:

**All T are F.  
Some T are C.  
Some C are F.**

As a Venn diagram would show, this argument is valid. Using the principle of charity, then, we can now say that the original argument is valid.

- **Arguments Containing Propositions in Need of Parameters**

Sometimes a sentence can look like it expresses a singular proposition when it does not. For example, consider the claim: "Socrates always wins the argument." Expressing the content of this sentence as a standard form categorical proposition is clearly a challenge. It has no standard quantifier; nor does it have a standard copula (the "are" that connects S to P); and it looks like a singular proposition. Perhaps the only thing crystal clear is that this is an assertion in the affirmative mode. So then, how do we proceed?

At this point, it is helpful to introduce the notion of a **parameter**. A parameter is best explained as a general class term such as "times," "places," "instances," "occasions," "cases," etc. that help to make it clear what the subject and the predicate classes are in an expression. As well, the inclusion of such parameters makes it easier to add the standard form copula "are" to the translation. Consider our example, "Socrates always wins the argument." What are the classes involved here?

Judging from the fact that "always" is a temporal term, we are led to think of the class of times. So we will want to add the temporal parameter, "times" to designate the classes involved in this expression. Well what sort of times? Letting the sense of the expression guide us, it seems reasonable to think of the times here as those times that Socrates engages in an argument, and those times that he wins an argument. And given the fact that "always" suggests "all times" it is reasonable to translate "Socrates always wins the argument" as "All times that Socrates engages in an argument are times that Socrates wins an argument." Other ways of expressing this are equally acceptable, such as: "All times Socrates argues are times he wins." In either case, we have a translation of the original expression into a standard form A proposition.

Besides "times," a common parameter is "places." Consider the old adage, "Home is where the heart is." Certainly this is no standard form categorical proposition. But it can be translated into a standard form if we use parameters. The sense of the assertion suggests "place" as a parameter. What suggests the parameter of "place" is the word "where." So we might translate the expression as follows: "All places that a person calls home are places where that person's heart is." Now we have a standard form A proposition.

We must not be misled into thinking that existential words always require a certain kind of parameter. For example, in the case just mentioned "where" correctly led us to think that a parameter of place is appropriate. But consider the following case, "Where there is a will there is a way." Here the word "where" is best translated as a temporal parameter. Also note that "times" is not the only temporal parameter. Indeed in this case, the sense of the expression suggests something like "occasions." So a reasonable translation of our expression might look something like this: "All occasions in which a person has the will to do x are occasions in which it is possible for that person to do x."

When it comes to translating arguments into standard form with the use of parameters, we must keep consistency in mind. If we adopt a existential parameter, we must use it consistently throughout the translation. Consider the following argument in ordinary language.

*Socrates always wins the argument.  
So of course he won, whenever he argued with Protagoras.*

We have already seen that the first premise can be translated with a temporal parameter as follows: "All times that Socrates engages in an argument are times when he wins an argument." Now we need to translate the second premise that asserts that he had an argument with Protagoras. So how do we render, "whenever he argued with Protagoras?" The sense of this assertion suggests "times that Socrates engages in an argument with Protagoras." So we might translate the second premise as, "All times that Socrates argues with Protagoras are times

when Socrates engages in an argument.” And the conclusion would be, “All times when Socrates argues with Protagoras are times when Socrates wins an argument.” We could represent this argument as follows:

*All E are W.*

*All P are E.*

*Therefore, All P are W.*

Now we have a standard form categorical syllogism of the form AAA-1. Now we are ready to test it with a Venn Diagram. Is it valid?

In the exercises to follow we will have more time to practice translations that require the addition of parameters.

- **Arguments Containing Conditional Propositions**

Many arguments in ordinary language are expressed in conditional sentences. A conditional sentence has an “If...then...” form. The “if” part of a conditional expression is called the antecedent and the “then” part, is called the consequent of the conditional. In this conditional relation between antecedent and consequent, the antecedent is the sufficient condition and the consequent is the necessary condition. This should be clear in the following conditional proposition: “If fire is present, then oxygen is present.” The presence of fire is a sufficient condition for the presence of oxygen, but the presence of oxygen is a necessary condition for the presence of fire.

So how do we express such conditional propositions as categorical propositions?

**The general rule is that we express conditionals as universal categorical propositions with the antecedent as the S-term and the consequent as the P-term.**

For example, we translate the proposition, “If I go to the movies, then I will see Jane” as follows: “All times that I go to the movies are times that I will see Jane.” And we can do the same for a conditional in the negative mode. For example, “If I go to the races I will not see Jane” as “No times that I go to the races are times that I will see Jane.” (Notice that we had to add a temporal parameter to make this translation.)

Many times conditional propositions are expressed without the use of “if” or “then.” For example, we might say, “To make a good grade in this class it is necessary that you attend class.” Clearly the sense of this assertion is that class attendance is necessary for making a good grade. This could be expressed as follows: “All students who make a good grade in the class are students who attend this class.” This is quite a different assertion than the following: “All you have to do to make a good grade in this class is to attend.” Now class attendance is made a sufficient condition for making a good grade. We would accordingly translate it as follows: “All students who attend class are students who make a good grade.”

**The general rule is that we translate a sufficient condition as the S-term and a necessary condition as the P-term of a universal categorical proposition.**

Sometimes “if/then” propositions are expressed with the term “only” or “none but,” “no one except” and similar phrases. When such phrases modify plural nouns or pronouns, they are called **exclusive propositions**. For example, we might say, “Only logic students can attend the party.” This sets up a necessary condition for attending the party. And what is this? Well, being a logic student is what is necessary, and if you are not a logic student then you are excluded from the party. Recalling that necessary conditions go in the predicate position of our standard form categorical proposition, we can translate this proposition as, “All people who can attend the party are logic students.”

Please note that it does not matter where the “only” occurs in the syntax of the exclusive proposition, what immediately follows it will be the necessary condition and hence the P-term of the categorical translation. We could

have said, for example, “You can attend the party only if you are a logic student,” and this would have the same translation.

**The general rule is that the term that follows exclusive words such as “only,” becomes the P-term in a universal categorical proposition.**

“You can attend the party only if you are a logic student,” is quite different from the following: “You can attend the party if you are a logic student.” Leaving out the word “only” makes being a logic student the sufficient condition for attending. So we would translate this proposition as follows: “All students of logic are people who can attend the party.”

Consider the following argument:

*Only logic students can attend the party. John can attend the party.  
Therefore John is a logic student.*

The first premise of this argument should give us no trouble. It should be translated as, “All P are L” (P=people who can attend the party; L=logic students). The second premise is a singular proposition. Accordingly, we must translate it as a universal proposition. So the second premise will be, “All J are P.” (J=the single-member class, John). The same thing is true of the conclusion. It can be translated as a universal proposition. The categorical translation of the argument then would look like this:

*All P is L.  
All J is P.  
Therefore, All J is L.*

This is an AAA-1 standard form categorical syllogism and a Venn Diagram will show that it is valid.

There is one other use of “only” that we need to address. Sometimes we say such things as, “**The only** candidates that we will consider are Harvard graduates.” Here what comes after “the only” is the sufficient condition in an “if/then” proposition. So it would be translated as follows: “If someone is a candidate that we will consider then he or she is a Harvard graduate.” And when translating this into categorical form, we get: “All candidates to be considered are Harvard graduates,” that is, “All C are H.” And don’t let syntax confuse you. It does not matter where “the only” occurs in the sentence. For example, “Harvard graduates are the only candidates that will be considered.” The proper translation of the proposition is the same as the previous expression, namely, “All C are H.”

Sometimes arguments in ordinary language involve “**either/or**” propositions. These expressions can be expressed as conditionals, that is, as “if/then” propositions. The general formula for translating “either/or” into “if/then” is as follows: “Either P or S”= “If not-P then S,” or “If not-S then P.” So if someone asserts, “I will either win or lose the game,” this could be expressed as, “If I do not lose, then I will win.” Of course such an assertion could be false, since it may be possible to tie. Nevertheless, this “either/or” proposition could be expressed as follows: “All times that I do not win are times that I lose” or “All times that I do not lose are times that I win.” Symbolically these formulations would be expressed as follows: “All non-W are L,” or “All non-L are W.”

Now consider this argument:

*When we play State we either win or lose. We have never lost to State.  
Therefore we always beat State.*

This certainly seems like a valid argument. Let’s see if we can translate it into standard form. The first proposition can be expressed as the following “either/or” proposition: “We win when we play State or we lose when we play State.” Now we just learned that we are permitted to translate this “either/or” proposition as an “if/then” conditional

proposition. So we translate it as follows: If we do not win against State, then we lose.” This “if/then” proposition can then be translated into standard categorical form as follows: “No occasions on which we lose to State are occasions on which we win against State.” The second premise is “All occasions on which we play State are occasions on which we win against State.” Therefore, “No occasions on which we play State are occasions on which we lose to State.” This translation yields the following standard form categorical syllogism (L =occasions of losing to State; W=occasions of winning against State; P=occasions of playing against State):

*No L are W.*

*All P are W.*

*Therefore, No P are L.*

This argument is an EAE-2 standard form categorical syllogism. Is it valid?

#### **4. Arguments Containing Exceptive Propositions**

A proposition that contains either the explicit words “all except,” “all but,” or some synonymous expression, is called an **exceptive proposition**. The following is an example: “All students except logic students can go to the party.” When we try to discern the propositional content of this expression we see that it says more than the simple conditional, “If you are a logic student you cannot go to the party.” It also asserts that if you are not a logic student then you can go to the party. In other words, an exceptive proposition is really a conjunction of two separate propositions jointly asserted in the same sentence.

We also count such propositions that involve phrases like “almost all,” “not quite all,” or “all but a few” as exceptive propositions, and thus also as expressing a conjunction of two separate propositions. Consider this assertion: “Almost all of our logic students did well on the LSAT.” Had we begun this assertion with “all but a few,” it would have had the same content. What is asserted is that “Some logic students did well on the LSAT” and that “Some logic students did not do well on the LSAT.”

Since exceptive propositions are really two propositions, can we translate syllogisms that involve them into standard form? Not as such. What we can do is to take arguments with exceptive propositions as expressing more than one syllogism. For example if we have an argument with an exceptive premise, and the other premise and the conclusion as standard form categorical propositions, we can translate that argument into two syllogisms. One syllogism will express one of the propositions expressed by the exceptive premise, and the other syllogism will have as a premise the other expression of the exceptive proposition. Consider the following argument:

*The students who did well on the LSAT are happy.*

*Not quite all of the logic students did well on the LSAT.*

*Therefore, some of the logic students are happy.*

The second premise in the argument is an exceptive premise. It makes two separate assertions, namely, “Some logic students did well on the LSAT” and “Some logic students did not do well on the LSAT. So we have two separate syllogisms here. They are as follows: (W=students who did well on the LSAT; H=students who are happy; and L=logicians.)

*#1*

*All W are H.*

*Some L are W.*

*Therefore, Some L are H.*

#2

*All W are H.*

*Some L are not W.*

*Therefore, Some L are H.*

Syllogism #1 is an AII-1 standard form categorical syllogism and is valid. Syllogism #2 is an AOI-1 standard form categorical syllogism and is not valid. We can see that the second syllogism is not valid, for it commits the fallacy of illicit quality (We cannot draw an affirmative conclusion when the argument has a negative premise.)

So one of these arguments is valid and one is invalid. So is the original argument with the exceptive premise valid? Again using the principle of charity, we say that the original argument is valid because the conclusions of both syllogisms are the same, and the exceptive premise includes the proposition that is expressed in the valid argument. That is, if the exceptive premise is true, then it is also true that “Some L are W.” And if “Some L are W” is true and it is also true that “All W are H” then it must be true that “Some L are H.”

If the conclusion of an argument is an exceptive proposition, then such an argument can never be valid. This is clear because the same premises cannot imply both of the propositions that are asserted in the exceptive proposition. For example, if the conclusion of an argument were, “Almost all the logic students did well on the LSAT” that conclusion would assert two propositions, namely, “Some logic students did well on the LSAT” and the proposition “Some logic students did not do well on the LSAT.” The two conclusions are independent assertions; hence both assertions cannot be implied by the same premises.

If we have a syllogism that has two exceptive premises, and a standard form categorical proposition as its conclusion, all of the various possible arguments will have to be expressed and evaluated. If any of these arguments is valid, the original argument is valid.

- **Enthymemes and Sorites**

Anytime that we are analyzing arguments, we must remember that they can be expressed without making all of their components explicit. This is also true in categorical logic. When a premise or a conclusion seems too obvious to mention, we can simply express the argument and leave this obvious component unexpressed. We have called such arguments **enthymemes**.

Enthymemes can occur in categorical logic. For example, “All dogs are mammals since all dogs are warm-blooded.” What is left out here is the implicit and obvious major premise that “all warm-blooded animals are mammals.” Once this implicit element is made explicit, we see that we have a standard form categorical syllogism of the valid form AAA-1: (W=warm-blooded; M=mammals; and D=dogs.)

*All W are M.*

*All D are W.*

*Therefore, all D are M.*

Other enthymemes leave out other components. A correct translation of an enthymeme into a standard form categorical syllogism must make these unstated components explicit.

As well, categorical syllogisms can form chains. We have called these chains of arguments **sorites**. The general form of such a sorites is as follows: the conclusion of one syllogism becomes the major premise of a new argument, whose conclusion can then become the major premise of yet another argument, and so on. If a sorites is to be valid, each of its component arguments must also be valid.

The following is an example of a valid sorites:

*All cats are independent.  
All tigers are cats.  
Therefore, all tigers are independent.  
Some wild beasts are tigers.  
Therefore some wild beasts are independent.*

Well, as you might have guessed, it is time to move on to our exercises. In the exercises for this chapter, you will be given a set of passages and your task will be to extract and symbolize the argument (s), if any, that (is) are expressed in these passages, and then to test the argument(s) for validity with Venn diagrams.

# Study Guide

## Chapter 6-Categorical Arguments

**Argument Form:** The form of an argument consists of its MOOD + FIGURE

- MOOD: the arrangement of AEIO sentences; for example AEO is the mood of a syllogism with an A sentence at the major premise, an E sentence as the minor premise and O sentence as the conclusion.
- FIGURE:

Figure #1 The Middle Term is in Subject of the Major and Predicate of the Minor (SP)

Figure #2 The Middle Term is in Predicate of both Major and Minor Premise (PP)

Figure #3 The Middle Term is in Subject of both the Major and Minor Premise (SS)

Figure #4 The Middle Term is in Predicate of the Major and the subject of the Minor (PS)

### Categorical Fallacies

**Illicit Terms:** A valid standard form categorical syllogism must contain exactly, and only, three class terms.

**Illicit Minor:** In a valid standard form categorical syllogism, if the minor term is distributed in the conclusion, it must be distributed in the minor premise.

**Illicit Major:** In a valid standard form categorical syllogism, if the major term is distributed in the conclusion, it must be distributed in the major premise.

**Illicit Middle:** In a valid standard form categorical syllogism, the middle term must be distributed in at least one premise.

**Illicit Quality:** In a valid standard form categorical syllogism, the conclusion must be negative if there is a negative premise.

**The Existential Fallacy:** In a valid standard form categorical syllogism, the conclusion cannot be existential if both premises are universal.

**Two Negatives:** A valid standard form categorical syllogism can't have two negative premises.

### The Three Step Venn diagram Test for Validity

Step One: Draw three interlocking circles and label each one with an uppercase letter designating one of the classes in the argument being tested.

Step Two: Diagram the major and the minor premises but never diagram the conclusion. It does not matter which premise you diagram first, unless one of the premises is an existential proposition and the other one is a universal proposition. In this case, always diagram the universal proposition first

Step Three: Examine the diagram to see if it contains a diagram of the conclusion of the syllogism you are testing. If it does, then the argument is valid; if it doesn't, the argument is invalid.

#### Valid Forms:

Figure 1 AAA EAE AII EIO;

Figure 2 EAE AEE EIO AOO

Figure 3 IAI AII OAO EIO

Figure 4 AEE IAI EIO

#### Traditionally Valid Forms

Figure 1: AAI; EAO

Figure 2: AEO; EAO

Figure 3: AAI; EAO

Figure 4: AEO; EAO; AAI