

Part II Categorical Logic

Chapter 5 – Categorical Sentence Forms

5.1 Introduction

It was Aristotle (384-322 BCE), the most famous student of Plato (428-347 BCE), who was the first to develop a formal system of logic. Aristotle's followers gathered his writings on logic and compiled them into what they called the *Organon*, which is a word that means "instrument." And this is exactly how Aristotle conceived of logic, as an instrument for the scientific and philosophical investigation of reality.

For Aristotle, reality is organized in *categories or classes*. Such classes have members and Aristotle thought knowledge of reality consisted of true sentences about these categories and their members. Such a sentence would be, for example, "All human beings are mortal."

What makes this system of logic formal is that, according to Aristotle, every true/false sentence, regardless of its content, can be expressed in one of a small number of basic sentential forms. He was aware of the fact if we could reduce all true/false sentences to these basic forms or types, then it would be much easier to manage the task of comprehending the ways in which these sentences are related in arguments. For example, consider the following classic valid argument:

All human beings are mortal.
Socrates is a human being.
Therefore Socrates is mortal.

Aristotle realized that the conclusion of this argument is related to its premises formally, that is, without respect to the fact that the sentences are about mortality, human beings, and Socrates. That is, he came to see that all arguments of this form are equally valid. To see this, consistently substitute anything you like for the categories of human beings, mortal and Socrates. (You might find it odd to think of Socrates as a category, but for Aristotle it was indeed a class, although it has only one member. We will discuss this issue more fully later.) Consider the following argument that has exactly the same form that the argument above has, and certainly is valid.

All dogs are mammals.
Fluffy is a dog.
Therefore, Fluffy is a mammal.

Now if there are only a few sentential forms, then there will be a small number of possible combinations of such sentences, that is, a small number of possible argument forms. Hence it will be easy to see, by a process of substitution, which of the thousands of arguments with their various and sundry contents are valid and which are not. This is because what makes the valid ones valid has nothing to do with their content. Rather what makes an

argument valid or not is simply a matter of the formal relation that exists between its premises and its conclusion. (Recall, an argument is valid if it is impossible for the conclusion to be false if its premises were true, and invalid otherwise.)

What an enormously important breakthrough in the study of logic. With Aristotle's ground-breaking insight, it became easier to distinguish between good reasoning and bad reasoning. To honor Aristotle's contribution, this system of logic is often referred to as "Aristotelian Logic."

In this chapter, and in the next, we will explore Aristotle's system of categorical logic in detail. What we will discover is that while it is a powerful system of logic, it does have some limitations. We will begin to see this especially when we come to the task of translating ordinary language into categorical sentences. As we will find out, translating from ordinary language into categorical form is not always easy to do. Indeed, sometimes it seems as though putting ordinary language into categorical form requires too much forcing, and perhaps even some distortion of meaning. This is so because our ordinary language is richer in its capacity to mark distinctions than the forms of categorical language can capture. It is this fact that finally gave rise to the development of different and more powerful logical systems. We will study these systems later when we get to sentential logic and predicate logic.

Now, we must turn to the first issue in our exploration of categorical logic, namely, to the formulation of the basic sentences in this formal system. In the next chapter, we will turn to the study of categorical arguments. Along the way, we will deal with the issue of translation, that is, with the issue of translating ordinary language into the language of categorical logic. Our discussion of translation in this chapter, and in the next, should prepare us for exploring more sophisticated formal systems.

5.2 Standard form Sentences

In the system of categorical logic, there are four standard form sentences, and **only four**. You mean everything that we can assert can be translated into one of these four forms? **That's right; there are only four kinds of sentences, only four**. Now don't forget, there are thousands of different sentences, if we differentiate them in terms of their content, but only four if we distinguish them in terms of their form.

So what are these four standard form categorical sentences? Well, here they are:

1. All S is P
2. No S is P.
3. Some S is P
4. Some S is not P

In the statements of these sentential forms, the "S" stands for the subject of the sentence, and the "P" stands for the predicate of the sentence. In case your grammar is a little rusty, let me refresh your memory about what subjects and predicates are. It might help to say that a predicate is what is said of a subject and the subject is what something is said of. Clearly the two terms are defined in relation to each other. To be more precise, however, we must state that relation in terms of categories or classes. So let's be clear that both the subject and the predicate terms in standard form categorical sentences are class terms (but notice again, classes may have only one member; and to complicate this a little more, a class may have no members at all; but we will get to this presently). In all of these sentential forms an sentence is made to the effect that the class or members of the class, designated by the subject

class term is either included in or excluded from the class, or members of the class, that is symbolized by the predicate term.

Clearly then, the sentence, “All human beings are mortal,” can be translated into the first standard form categorical sentence, “All S is P.” As we might state this: “All members of the class of human beings are also members of the class of mortal beings.” This statement captures the sense of the original sentence, as does its symbolic expression, “All S is P.”

Before we go on to consider the other standard form sentences, let's adopt the standard designation of each of these sentences. This will make it easier to refer to them. We will now designate each sentence with a capital letter. These letters are A, E, I, and O. The four standard form sentences then are as follows:

A: “All S is P”
E: “No S is P”
I: “Some S is P”
O: “Some S is not P”

So there you have it, categorical logic has four, and only four, standard form sentences. And it is a matter of convention, that is, a matter of convenience, that we refer to them with the letters, A, E, I, and O respectively.

From this, it should be clear that our example above, namely, “All human beings are mortal,” is an A sentence. And it is easy to think of examples of other A sentences.

Now let us consider the E sentence: “No human beings are pigs.” Clearly this means (literally speaking; we will leave metaphor out of the picture at this point) that no member of the class of human beings is also a member of the class of pigs. As opposed to the A sentence, which is an sentence that the class designated by its S-term is included within the class designated by its P-term, the E sentence is an sentence that the class designated by its S-term is excluded from the class designated its P-term.

When a sentence asserts a relation of class inclusion (actually it is speakers who assert things *with* sentences, but for convenience, we will adopt this shorthand way of referring to sentences as asserting something) we will say that it has an **affirmative quality**. By the same token, when a sentence asserts a relation of class exclusion, we will say that it has a negative quality. Accordingly, the A sentence has an affirmative quality while the E sentence has a **negative quality**.

In categorical logic, we must sharply distinguish the “quality” of a sentence from its “quantity.” Just as there are two qualities that sentences can have, namely, affirmative and negative, they may also have two different quantities. These two quantities are commonly designated with the terms “universal” and “existential.” Every standard form categorical sentence is said to be either universal or existential in quantity. Both “all” and “no” are universal quantifiers since they refer to all of the members of the class of S in the A and E sentences: “All S is P” and in “No S is P”. “Some”, which means “there exists at least one” is an existential quantifier.

OK, let's re-group. So far it should be clear that of the four Standard form categorical sentences, A and E sentences are both **universal in quantity** and the A sentence is **affirmative in quality** while the E sentence is **negative** in quality. What makes them both universal is that in both cases, the S-term of each sentence is talking about every member of the class it designates. What makes the A sentence affirmative is the fact that it asserts that the class

designated by its S-term, in part or as a whole, is included within the class designated by its P-term. What makes the E sentence negative is that the class designated by its S-term, in part or as a whole, is excluded from the class designated by its P-term. (It is probably the case that the A designation derives from the Latin term for “I affirm,” just as the E designation derives from the Latin term for “I deny,” which when pronounced sounds like NEE-go, hence the E.)

In our two examples above, the S-term is the class of human beings. In the first case, the A sentence, “All human beings are mortal” is a **universal affirmative** sentence because every member of the class designated by the S-term, “human beings” is asserted to be included within the class designated by the term “mortal (beings).” In the second case, the E sentence, “No human beings are pigs” is a **universal negative** sentence because every member of the class designated by the S-term, “human beings,” is asserted as excluded from the class designated by the P-term, “pigs.”

So much then for the A and the E sentences. Let’s turn to the two others, namely, the I sentence and the O sentence. Like their cousins the A and the E, these two sentences have a common quantity and a different quality. Consider the quality of the two sentences first. The I sentence is affirmative, the O sentence is negative. The same reasoning holds with regard to the quality of these two sentences as in the case of their universal counterparts. The I sentence, “Some S is P” is affirmative in quality because it asserts that the class designated by its S-term, in part or as a whole, is included within the class designated by its P-term. Similarly, the quality of the O sentence, “Some S is not P” is negative because it asserts that the class designated by its S-term, in part or as a whole, is excluded from the class designated by its P-term.

If we say that a sentence is universal in quantity because its S-term refers to all of the members of the class that it designates, we are going to need a term to capture the case in which the S-term does not refer to all of its members. That is, we need a term to express the case in which the S-term refers to only some (at least one) of the members of the class it designates. The term some logicians have adopted to capture the quantity of “some” is “existential.” Accordingly, it is said that the I sentence and the O sentence are both existential in quantity. Often, however, the term “existential” is used instead of the term “existential.” I prefer the term “existential” to “existential” since to say that “Some S is P” or that “Some S is not P” is just to say that there **exists** at least one member of the class S.

In sum then, here is where we are. I and O sentences are both existential in quantity and the I sentence is affirmative in quality and the O sentence is negative in quality. Every standard form categorical sentence has either a universal or an existential quantifier and is either affirmative or negative in quality.

The text box below shows the various combinations of qualities and quantities that characterized the four standard form categorical sentences.

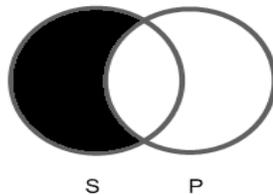
| |
|---|
| <p style="text-align: center;">Quantity Universal: A • E; Existential: I • O Quality: Affirmative: A • I; Negative: E • O</p> |
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Categorical logic lends itself very nicely to visual representation. This is so because it involves class inclusion and exclusion. Classes can easily be represented with **Venn Diagrams**. We can represent all of the standard form categorical sentences with two intersecting circles, each representing a separate class, one the S-class and the other the P-Class.

DIAGRAMMING A, E, I, and O SENTENCES:

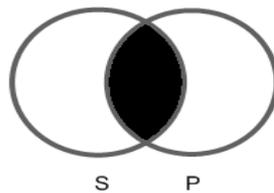
In the case of the A sentence, we represent the claim that all of S is included within P ("All S is P") by emptying that section of the circle representing the S-class that is outside of the circle representing the P-class. That is, we empty the section of the interlocking circles that is excluded from P by shading it in black, as indicated below. To blacken a section is to erase it. The diagram shows that all of the class of S that is left (after the blackening process) is contained wholly within the class P.

THE A SENTENCE



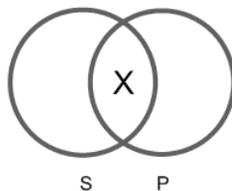
We represent the E sentence, "No S is P" in a similar way. To represent the sentence that all of S is excluded from all of P, we empty the section of the interlocking circles that is common by shading it in black, as indicated below. Again, to blacken a section of the diagram is to erase it. Now clearly all of S that remains is excluded from P.

THE E SENTENCE



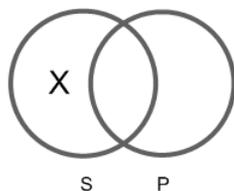
We represent the I and O sentences differently. We represent the I sentence, that is, the sentence that "Some S is P" by showing that the class S has at least one member and that this member of S is also a member of the class of P. Remember, "Some S is P" is to be read as: "There eXists and S..." We can show this by placing an "X" in the area that is common to both S and P as follows:

THE I SENTENCE



Finally, we represent an O sentence, that is, the sentence that "Some S is not P" by showing that the class S has at least one member and that this member is not a member of the class of P. We do this by placing an "x" the area of S that is not in the area of P as follows:

THE O SENTENCE



When I have talked so far about whether a term refers to all or to only some (at least one) member of the class it designates, that is, when I have talked about whether a sentence is affirmative or negative, I have only mentioned the range of the S-term. (It can range over the whole or over a part of the class it designates.) I have not mentioned the range of the P-term. We need to do this and to make it clear that both the S-term and the P-term in a standard form categorical sentence can refer to either all or part of a class. But first I need to introduce the concept of distribution and let you know what distribution means in this context. With this definition, we will be able to see that the every S-term and every P-term in every standard form categorical sentence is either **distributed** or **undistributed**

So here is our definition of “distribution”: Any S-term or P-term is distributed if and only if it refers to all of the members of the class that it designates, and any S-term or P-term is undistributed if and only if it refers to only some (at least one) member of the class that it designates.

It should be obvious that in both the universal sentences, A and E, the S-term is distributed. Clearly in the sentences “all human beings...” and “no human beings...” the S-term leaves no member out of the class it designates.

Similarly, in both the existential sentences, I and O, (“Some S are...” and “Some S are not...””) the S-term is referring to only some (at least one) members of the class it designates. Hence, in the I and O sentences, the S-term is undistributed.

So where does this leave us with regard to the P-terms of our Standard form sentences? Let’s take the universal sentences first. In the A sentence, “All human beings are mortal” the sentence is not intended to refer to all of the members of the class designated by its P-term, “mortal beings.” We say then that the P-term in an A sentence is undistributed.

In the E sentence, “No human beings are pigs” the intention is to exclude every member of the class of human beings from the entire class (“pigs”) designated by the P-term of this sentence. Accordingly, we say that the P-term in an E sentence is distributed.

In the case of the existential sentences I and O, both of which have undistributed S-terms, what do we say of their respective P-terms? In the case of an I sentence, for example, “Some cats are pets” we are clearly not talking about the entire class of pets. Hence the P-term in this sentence, which designates the class of “pets,” is not intended to refer to every member of that class. The whole class of cats can “fit” into the class of “pets” without exhausting the class of “pets.” There is room for dogs in that class, for example. So we say that the P-term in an I sentence is undistributed.

In the case of an O sentence, for example, “Some cats are not pets” we are clearly talking about the entire class of pets. That is, some cats, feral and other kinds, fall completely outside the entire class of pets. There is, in other words no room in the class of pets for such creatures. So the P-term in an O sentence is distributed.

Now let’s notice some parallels in summary. First, both universal sentences, A and E, always distribute their S-terms. The S-terms in both existential sentences, I and O, are undistributed. The P-terms in both affirmative sentences, A and I, are undistributed, and in both negative sentences, E and O, the P-term is distributed.

Here is a more graphic summary:

Distribution Table

| |
|--|
| A: S-term distributed; P-term undistributed (DU) E: both terms distributed (DD) I: both terms undistributed (UU) O: S-term undistributed; P-term distributed (UD) |
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5.3 More Complex Translations

It is quite obvious that in ordinary modes of conversation, we do not tend to talk or write in Standard form categorical sentences. Indeed, there is something almost unnatural, or at least stilted, about such Standard form categorical expression. This is a problem for categorical logical analysis; however, since our method of assessing the validity of categorical arguments is going to depend on how accurately we translate ordinary expressions into Standard form categorical sentences.

Fortunately, many, perhaps even most, but certainly not all, of our ordinary true/false sentences can be reformulated into standard form categorical sentences. A few hints will help in this process. For example, some sentences do not have explicit quantifiers, or else they are expressed in non-standard ways. When we find such expressions, we must interpret which quantifier is intended and supply it. Let's see how this might work.

Consider the following sentence: “Cheetahs run fast.” Clearly, this sentence is not in standard categorical form. A little reflection, however, will make it clear that the implicit quantifier here is universal. That is, the proper translation of this sentence should be "All C are R."

Matters would be quite different if the sentence had been: “Cheetahs were recently added to the zoo’s collection.” In this case it is implausible to think that the zoo now has every Cheetah in its collection. So clearly the proper translation of this expression should be: “Some Cheetahs are animals recently added to the zoo’s collection.”

The same sort of distinction exists between the following two sentences: (1) “Roses have thorns” and (2) “Roses are taking over my flower bed.” In the first case, we are clearly talking about “all roses” whereas in the second we are referring to “some roses.”

In determining the quantifier to use in cases where no quantifier is specified, do not be misled into thinking that the articles “the” or “a” or “an” settle the issue. They do not. For example, “The Cheetah is endangered” refers to “all Cheetahs,” and hence is universal in quantity. Just as clearly, “The Cheetah escaped” refers to only one, and hence is existential in quantity. And similarly, “A mind is a terrible thing to waste” refers to “all minds,” and hence is universal in quantity, just as “a great mind was lost when he died” refers to only one mind, hence is existential in quantity.

Sometimes qualifiers and quantifiers are embedded in an ordinary expression and this may pose a challenge for translation. Consider this example: "My students are not all well motivated." Even though this sentence has "all" in it, there is no plausible way to translate it as an A sentence. The reason for this is that its quality is clearly negative, and the quality of an A sentence is affirmative. As well, the sense of the sentence seems not to be universal, since what seems to be intended in the claim that "not all..." like "not every" is an alternative way of expressing "some are not..." For example, the sentence "Not all Americans vote" clearly means: "Some Americans do not vote." Accordingly the translation of our sentence above should be the following O sentence: "Some students in my class are not well motivated."

And consider one more thing regarding negative qualifiers. Had the expression above been "not any" instead of "not every" or "not all" the translation would have required a different quantifier. Clearly "not any" is a universal claim, and could best be translated as: "No student in my class is well motivated."

We might formulate a general guideline regarding quantifier and qualifier translation as follows: absent qualifiers and quantifiers should be supplied and non-standard qualifiers and quantifiers should be translated as standard quantifiers. "A few," for example, should be translated as "some," and "not everyone" should be translated as "some are not," and "anyone" should be translated as "all," and so forth.

Now there are other such general guidelines for good translation. Below are a few. Keep in mind, however, that good translations must preserve the sense of the original sentences that are being translated. Nevertheless, even the most creative art can make use of some guidelines. So, again, here are a few:

1. Translate an sentence that contains **exclusive terms** such as "only" or "only if," "none but," "except," "none except," and so forth, regardless of where these terms of exclusion occur in the sentence, as a universal affirmative (A) sentence and make what follows the term of exclusion the predicate term of that sentence. For example, translate, "None but the brave are rewarded" as "All R (people rewarded) is B."
2. Translate an sentence that contains "**the only**" as an A sentence. For example, translate "**The only** survivors were children," as "All S is C."
3. Translate an sentence that contains "**if**," regardless of where it occurs in the sentence, as a universal (affirmative or negative) sentence, and make what follows the "if" the subject term of that sentence, e.g. translate "You can go **if** you study" as "All S (times you study) are G (times you can go) or "You cannot go if you sleep late" as "No S (times you sleep late) are G (times you can go." Remember, there is a very big difference between "if" and "only if." When an sentence contains "only if" take this as stating a necessary condition and translate it as follows; translate this by making the only if statement into an A sentence with what immediately follows the "only if" as the predicate term of this A sentence. For example: "You may go to the movies only if you do your homework" should be translated as All M (times you can go to the moves) are H (times you do your homework).
4. When "**not**" is contained in an "all" sentence, as in "All S are not P," translate the sentence as an "E" sentence ("No S is P.")
5. Translate sentences with non-standard verbs into the verb "**to be**." For example, "Good logic students study hard," would be translated as "All G (good logic students) are S (students who study hard).
6. When a term in a sentence does not contain a noun, then it must be supplied. For example, "Some people smoke" should be translated as "Some P (people) are S (smokers)."

7. Sentences with singular proper names are complicated. For now we will say that such sentences should be translated as universal sentences. For example, "Socrates is mortal" is translated as the A sentence "All S are M." As well, "Socrates is not mortal" should be translated as the E sentence "No S are M." (Admittedly, most, but not all, singular sentences seem to carry existential import so this practice of translating singular sentences into universal sentences needs qualification when it comes to assessing the validity of arguments containing singular sentences. This is discussed in the next chapter.)

8. Sentences of existence, such as "there is" should be translated with the use of the existential quantifier "some". For example, "there is ice in the refrigerator" should be translated as "Some I is R."

In the next chapter, we will return to this issue of translation. As we will see, the problem of translating ordinary language into standard form categorical sentences is one of the biggest difficulties in the Aristotelian formal system, and one of the reasons that new systems of logic were developed later.

5.4 The Traditional Square of Opposition

Hopefully it is obvious to you that these four standard form categorical sentences bear some relation to one another regarding their truth values, at least when the class terms in these sentences are common. For instance, if we are talking about the class of human beings and the class of pigs, the fact that the E sentence involving these two classes, e.g., "No human beings are pigs" is true, means that we can immediately infer that its corresponding universal sentence, namely, the A sentence, "All human beings are pigs," cannot also be true, that is, must be false. As well, if the I sentence "Some human beings are pigs" is false, then the corresponding existential, namely, the O sentence, "Some human beings are not pigs" cannot also be false, that is, must be true.

We can organize these various **immediate inferences** in terms of what has been called a system of oppositions. There are three basic categories of these oppositions. The first category is that of **contradiction**. No doubt you recognize that people sometimes contradict themselves. (Actually the term literally means, "to speak against.") Perhaps, however, you do not know exactly what the charge that we have contradicted ourselves amounts to. Well in the language of categorical logic, we can now define the notion of contradiction more precisely. We will say that **two sentences are contradictories if and only if they both cannot be true and they both cannot be false**. This being true, then of course if a given sentence is true its corresponding contradictory sentence must be false, and if false, its corresponding contradictory sentence must be true.

In categorical logic A and O sentences are contradictories and E and I sentences are also contradictories. Knowing this allows us to make some very useful immediate inferences. If I know that "Some human beings are not pigs" is true, the sentence "All human beings are pigs" must be false. As well, if "Some cats are pets" is true, then its corresponding contradictory sentence, "No cats are pets" must be false. And we can go in the other direction also. If "No people are pigs" is true, then its corresponding contradictory sentence, "Some people are pigs," must be false. In general then, if we know that a given sentence is true then we know that its contradictory sentence is false, and vice versa.

As we might put this, immediate inferences of this sort constitute valid arguments. That is, if we assume an A sentence is true, then it must be the case that its corresponding sentence, the O sentence, is false. This is called an immediate inference because it is not mediated by any other premise. Immediate inferences involve only one premise and only one conclusion.

Contradiction, however, is not the only relation of opposition. Sentences can also be related to one another as **contraries**. **Two sentences are contraries if and only if they cannot both be true, though they could both be**

false. This relation exists only between the two universal sentences, the affirmative A sentence and the negative E sentence. Given that A and E sentences are contraries, if we know that an A sentence is true, then we also know that its corresponding E sentence must be false. And if we know that an E sentence is true, then we know that its contrary, the A sentence must be false. Contraries, to repeat, cannot both be true. But since they can both be false, knowing that an A or an E sentence is false, tells us nothing about the truth value of its contrary.

The same relation does not hold between the corresponding existential affirmative and negative sentences, namely, the I and the O sentences. These two existential sentences are said to be **sub-contraries**. **Two sentences are sub-contraries if both sentences cannot be false, even though both can be true.** So, if we know that “Some cats are not pets” is false, then we can immediately and validly infer that “Some cats are pets” is true. Again, however, the fact that the sentence “Some cats are pets” is true, does not imply that the sentence “Some cats are not pets” is false, for clearly both of these sentences are in fact true. (Please take note of the difference between the terms “infer” and “imply.” The premises in a valid argument imply its conclusion, but only people can infer, as well as imply something, that is, only people can make inferences. Premises do not infer their conclusions; they imply them, if valid. Persons make inferences, valid and invalid. There are no invalid implications.)

There is one more such opposition to consider. This is the relation called **sub-alternation**. This is a relation between a universal sentence and the corresponding existential sentence with the same quality (affirmative or negative). The relation of sub-alternation exists accordingly between an A and an I sentence and between an E and an O sentence. It is really quite an obvious relation when you think about it. If an A sentence is true, for example, “All human beings are mortal” then surely it must be true that “Some (at least one) human being is mortal.” And if “No human beings are pigs” is true, then it must be true that “Some (at least one) human being is not a pig” is also true. However, if we know that an A or an E sentence is false, we cannot immediately infer anything about its corresponding existential sentence. If “No cats are pets” is false, we do not immediately know that “Some cats are not pets” must also be false (or must be true), although as a matter of fact it happens to be true. And if we know that “All cats are pets” is false, we do not also immediately know that “Some cats are pets” must be false (or must be true), although as a matter of fact it happens to be true.

If we turn the relation of sub-alternation around, we can make a couple of more immediate inferences. If an I sentence is false, then surely its corresponding A sentence must be false. If it is false that “Some human beings are pigs” then surely it must be false that “All human beings are pigs.” And if “Some cats are not pets” is false, then surely, “No cats are pets” must also be false. However, nothing follows about the truth or falsity of their corresponding universal sentences if we know only that the existential sentence is true. From the fact that the I sentence “Some cats are pets” is true, nothing follows about the truth or falsity of the A sentence, “All cats are pets” which as a matter of fact is false. We could, however think of a case in which an I sentence is true and its corresponding A sentence is also, as a matter of fact, true. For example: “Some cats are mammals” is true, and so is “All cats are mammals.”

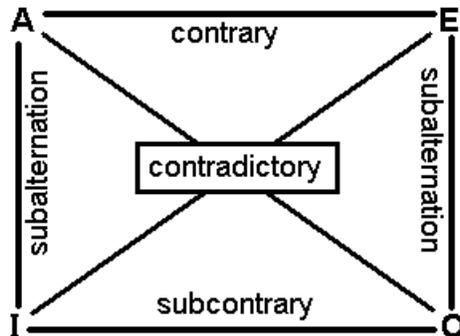
And the same can be said about the relation between the O and the E sentences. If the O sentence is false, then the corresponding E must also be false. However, if the O sentence is true, nothing follows about the truth or falsity of its corresponding E sentence, that is, it may be true or it may be false. Think of some examples that demonstrate this relation of sub-alternation.

We see then that:

- Contraries: cannot both be true, but both may be false
- Sub-Contraries: cannot both be false, but both may be true

- Contradictories: cannot both be false, cannot both be true
- Sub-Alternations: If the super is true, the sub must be true; if sub is false, super must be false

The image below represents the various relations of immediate inference in the traditional square of opposition.



The boxes below summarize the various immediate inferences in the traditional square of opposition:

**If A is true, then E is false, I is true, and O is false.
If A is false, then O is true, and E and I are unknown.**

**If E is true, then A is false, I is false and O is true.
If E is false, then I is true and A and O are unknown.**

**If O is true, then A is false and E and I are unknown.
If O is false, then A is true, E is false and I is true.**

**If I is true, then E is false and A and O are unknown.
If I is false, then A is false, E is true, and O is true.**

5.5 The Existential Assumption

However brilliant Aristotle was, he did overlook one important thing in the formulation of his system of categorical logic. What he did not consider is that classes can be empty. Aristotle assumed that every class has at least one member. We call this assumption, **the existential assumption**. Recognizing that this assumption is not necessarily true has enormous consequences for categorical logic, especially with regard to the immediate inferences that we have just been discussing.

Before we can say what the consequences are of making the existential assumption, we must say a word about the difference between the universal and the existential quantifiers. As we have said, the existential quantifier “some” is to be read as asserting, “There exists at least one.” So, for example, the sentence “Some S is P” is to be read as asserting that its subject class has at least one member. “Some flowers are roses” asserts that there is at least one

member of the class of flowers and that member is included also in the class of roses. Given the fact that existential sentences make an existential sentence, and given that classes can be empty, we must draw the following conclusion: Every existential sentence that has an S-term that designates an empty class is false. For example, consider the sentence, "Some Martians are kind." If we can grant that the class of Martians is empty, the sentence that there exists at least one member of that class that is kind must be false, for no members of that class exist. And the same can be said for the corresponding O sentence, "Some Martians are not kind." Both of the sentences are false, since they both assert that there are members of their respective subject classes, when these classes are empty.

Recall that in Aristotle's system the I and the O sentences are sub-contraries. Recall also that sub-contraries cannot both be false. As Aristotle reckoned, if we know that a given existential sentence is false we can validly and immediately infer that its corresponding sub-contrary is true. We now recognize that this immediate inference is valid only if we make the existential assumption. If we do not, that is, if the class designated by the subject term of an existential sentence is empty, both of these sentences are false, as we just pointed out in the case of the sentences about Martians.

The moral here is this: the immediate inference from the falsity of a given existential sentence to the truth of its corresponding sub-contrary does not hold unless the existential assumption is made.

Again, since it is obvious that some classes are empty, we must again augment the Aristotelian system of logic. Accordingly, we will say that the immediate inference from the falsity of a given I or O sentence to the truth of its corresponding sub-contrary is not a valid immediate inference, unless the existential assumption is made. Since that assumption is not normally made, we will say that normally any immediate inference from a given existential sentence to its sub-contrary is not valid.

When we turn to universal sentences, things are a little more puzzling. Consider the sentences, "All Martians are kind" and "No Martians are kind." Given that the class of Martians is empty are these two sentences true or false? Is there any sentence of existence in these two universal sentences? Logicians have agreed that the answer to this question is "no." If you have trouble seeing this, think about the following two sentences: "All the money in my wallet is yours" and "No money in my wallet is yours." Now suppose, as it is not hard to imagine, that I have no money in my wallet. In this case, that is, if the subject term here, namely the class of "money in my wallet" is empty, then both sentences seem to be true. If I give you all the money in my (empty) wallet and if I give you none of the money in my (empty) wallet, I will have given you the same amount of money, namely, no money.

The moral of these considerations is that universal sentences do not assert the existence of members of their subject classes. As we just demonstrated in the case of the class of "money in my wallet" when that class is empty, both an A sentence and its corresponding E sentence can be true. In Aristotle's system the A and the E sentences are contraries, which means that both cannot be true. We now see that the inference from the truth of a given universal sentence to the falsity of its corresponding contrary is not valid unless we make the existential assumption. So again, the immediate inference from the truth of a given A or E sentence to the falsity of its corresponding contrary is not a valid immediate inference, unless the existential assumption is made. Since that assumption is not normally made, we will say that normally any immediate inference from a given existential sentence to its contrary is not valid.

The immediate inferences from the truth of A and O sentences to the truth of their corresponding subalterns meets with a similar fate if the existential assumption is not made. It should be obvious that if "All Martians are kind" is true because its subject class is empty, it does not follow that its subaltern, "Some Martians are kind" would also have to be true. In this case, because the quantifier "some" asserts that some Martians exist, the subaltern of the A

sentence, its corresponding I sentence, can be false even when the corresponding A sentence is true. The same holds for the relation between the E sentence and its corresponding subaltern, the O sentence. Again the moral: the immediate inference from the truth of a given A or E sentence to the truth of its corresponding subaltern is not a valid immediate inference, unless the existential assumption is made. Since that assumption is not normally made, we will say that normally any immediate inference from a given existential sentence to its subaltern is not valid.

So what is left of Aristotle's system of immediate inferences once we abandon the existential assumption? With regard to the inferences involving contraries, sub-contraries, and subalterns, the answer is "nothing." But this is not the whole story. Even if we abandon the existential assumption, some immediate inferences in the Aristotelian system of categorical logic do remain valid. These are the valid immediate inferences that are involved in the opposition called contradiction. Given that the sentence "No Martians are kind" is true because there are no Martians, it is still the case that the corresponding contradictory I sentence, "Some Martians are kind" must be false since it asserts that the class of Martians has members when it does not. The same holds for the relation between the A and the O sentences. So we again arrive at a moral: the immediate inference from the truth of a given sentence to the falsity of its corresponding contradictory sentence, or from the falsity of a given sentence to the truth of its corresponding contradictory sentence, are valid immediate inferences, even if the existential assumption is not made.

It is almost time to put this material to the test. The exercises below should help you on your way in sorting out these various immediate inferences and in grasping firmly the traditional square of opposition and the import of making and not making the existential assumption. For convenience, we will call the square of opposition that does make the existential assumption "The Traditional Square of Opposition" and the square of opposition that does not make the existential assumption, "The Modern Square of Opposition." As well, inferences that are valid only if the existential assumption is made we will say are "traditionally valid".

5.6 Other Immediate Inferences

There are three other important kinds of immediate inferences that we need to consider. The first one, **conversion**, is the most obvious, for it is quite common to hear such expressions as, "All college teachers have college degrees but not conversely." What this means is that it is false that "All people who have college degrees are college teachers." Notice that second sentence simply turns the S and the P terms around. **In general, we say that the converse of a given sentence is obtained by exchanging the S-term and the P-term.** In the case we just mentioned, the converse of "All S is P" is "All P are S." And this is true of all of the Standard form categorical sentences. The converse of the E sentence "No S is P" is "No P are S," the converse of the I sentence "Some S is P" is "Some P are S," and the converse of the O sentence "Some S is not P" is "Some P are not S." We can also reverse, or double this procedure, e.g., the converse "No S is P" is "No P are S," and the converse of "No P are S" is "No S is P." Or as we might put this, the converse of the converse of "No S is P" is "No S is P." The same operations apply to all of the Standard form categorical sentences.

OK, so far this is easy enough. The slightly more difficult question that arises, however, has to do with which of these conversions are valid immediate inferences and which are not. But before we say which are and which are not, let us just pause and make sure we are clear about what an immediate inference is, and especially what a valid one is.

Since inferences can be valid (or invalid), and since these terms apply only to arguments, it follows that an immediate inference is an argument. (Did you notice that what we just said was itself an argument, a deductive one, and valid to boot?) Well, what makes such inferences "immediate" is that they are not "mediated" by some other

premise. In general we say that an immediate inference is one in which some given sentence implies some other corresponding sentence without the intervention of any other sentence. We have seen this already in, e.g. contrary sentences. That is, from the fact that an A sentence is true, it follows immediately that its corresponding contrary E sentence must be false. Such an immediate inference is a valid argument form because it accords with the following general rule that applies to contraries: If a given sentence (either an A or an E sentence) is true, then its corresponding contrary must be false. (We call such an inference a valid argument form, since the relation between the two sentences holds regardless of their existential contents.)

With this said, let's now see if we can determine which standard form categorical sentences can be validly converted and which cannot. We have already noted, an A sentence does not validly imply its converse. If we assume that an A sentence is true, we cannot validly infer that its converse is true. We have already seen that this is an invalid inference in the example above. Clearly, if it were true that "All college teachers have college degrees" is true (and of course even though most do, it is obviously true that some do not), the converse of this sentence would not have to be true as well. Hence we say that an A sentence does not validly imply its converse, or if you will, the immediate inference from an A sentence to its converse is invalid.

But quickly, we must qualify this observation. Intuitively, it should be obvious that from the truth of the sentence, "All college teachers have college degrees," it does follow that "Some people who have college degrees are college teachers" must also be true. This is a valid immediate inference. But notice that the second sentence not only reverses the S and P terms of the original A sentence, it also limits the quantity of the original A sentence, that is, it changes the quantity from universal to existential, from "all" to "some." If we make this additional change, then the immediate inference from the truth of its limited converse is valid. We can generalize this as follows: **the inference from an A sentence to its limited converse is valid, and without this limitation, the inference is invalid. But this inference is only traditionally valid (or valid by limitation since the inference is from a universal sentence (which does assert existence). If the S-class were empty this inference would not hold.**

Converting the other sentences is less qualified. The immediate inference from an E sentence to its converse is valid, and so is the immediate inference from an I sentence to its converse, but the immediate inference from an O sentence to its converse is not valid. This should be clear if you simply make some substitutions into our sentential forms. Clearly if the E sentence "No human beings are pigs" is true, then it must be true that "No pigs are human beings," and vice versa. Just as obviously, from the truth of the I sentence, "Some roses are flowers" it follows that "Some flowers are roses," is also, and must be, true. Just as surely, it should be obvious that if an O sentence, "Some cats are not pets" is true, it does not follow that "Some pets are not cats" must also be true.

To summarize once again: The immediate inference to the contrary of an E sentence and to the inference to the contrary of an I sentence are valid while the inference to the contrary of an O sentence is invalid. With regard to the A sentence, we say that while the inference to its unqualified contrary is invalid, the inference to its contrary qualified by limitation is valid.

The second of the three kinds of immediate inference we are looking at is called **obversion**. This one is easy in this respect: the inference from any of the four Standard form categorical sentences to its obverse is valid. What needs explanation here is the operation for deriving the obverse of a given sentence. As we just saw, the operation for obtaining the converse of a sentence is simply switching the S and the P terms. The operation for obtaining the obverse of a given sentence is a little more complicated and requires that we introduce a new concept, namely, the concept of a **complementary class**.

We will say that the complement of a given class is the class of everything outside of the given class. The complement, for example, of the class of cats is the class of everything that is not a cat. But we have to be careful about how this “not” is being used. Clearly dogs are not cats, that is, the class of dogs is outside the class of cats. But the class of dogs is not the complement of the class of cats, since the class of pigs is also outside the class of cats. To make this clear, we adopt the use of the sometimes grammatically awkward prefix, “non.” The complement of the class of cats is accordingly the class of “non-cats,” dogs, pigs and sealing wax included. As well, the class of the complement of the class of “non-cats” is the class of cats, the class of non-non-cats. Just as in the case of two negatives, two “nons” or a “non-non,” cancel each other out.

So now we are ready to say what the operation is for obtaining the obverse of a given sentence. This operation has two steps. **First**, change the quality of the given sentence, either from affirmative to negative or vice versa. That is, the obverse of any given sentence has the opposite quality of the given sentence. For example, the obverse of the E sentence is an A sentence and vice versa, and the obverse of an I sentence is an O sentence and vice versa. That is, in making this first step in obtaining the obverse of a given sentence, we change the quality of the given sentence but leave its quantity intact; if a sentence is universal, its obverse will still be universal; if its quantity is existential, its obverse remains existential.

The **second step** in deriving the obverse of a given sentence is to replace the P-term of that given sentence with its complement. So the obverse of an A sentence “All S is P” is the E sentence, “No S are non-P.” It should be clear that this is a valid immediate inference if you give these sentential forms some content. For example, it should be clear that the A sentence “All roses are flowers” is logically equivalent in truth value to the E sentence “No roses are non-flowers,” and vice versa. As well, you should be able to see that “Some S is P” logically implies its obverse, “Some S is not non-P” and that “Some S is not P” logically implies its obverse, “Some S is non-P”.

I need to point out here that it is also true that if a given sentence were false its obverse would also be false. As we might put this, every Standard form categorical sentence is logically equivalent to its obverse. Two sentences are logically equivalent if they have the same truth values and cannot have different truth values. Logically equivalent sentences assert the same thing, and hence one can be substituted for the other without changing the truth value of either. Since the immediate inference of the obverse of any given sentence is a valid inference, we say that any given sentence and its obverse are logically equivalent in truth value. (This equivalence relation also applies to the operation of conversion, with the exception of conversion by limitation. For example, “No S is P” is logically equivalent to its converse, “No P are S.” This is just another way of saying that if one is false the other is also false, and if one is true the other is also true. Just as clearly, if “Some P are S” is true we do not know that “All S is P” is also true. Consider this: “Some pets are cats” is not logically equivalent in truth value to the sentence, “All cats are pets.” In general we have established that E and I sentences are logically equivalent to their converses; and A, E, I, and O sentences are logically equivalent to their obverses.) When two sentences are logically equivalent we can substitute one sentence for the other without changing the truth value or meaning of either sentence.

The last kind of immediate inference we will consider is **contraposition**. In order to derive what is called the contrapositive of a given Standard form categorical sentence, all we have to do is follow two steps. **First**, we switch the S-term and the P-term around (that is, the S-term becomes the P-term and the P-term becomes the S-term.); **second**, we replace each of these class terms with its complement. (Actually there is no proper order to these two steps: you can first replace both the S-term and the P-term with their complements and then make the switch.). For example the contrapositive of the A sentence, “All S is P” is “All non-P are non-S.” (Notice that in deriving this contrapositive we leave the quality and the quantity unaltered.)

As a matter of fact the immediate inference from an A sentence to its contrapositive is valid, as we can see from the following example: if the sentence “All students are wisdom seekers” is true (or false), then surely its corresponding contrapositive, namely, “All non-wisdom seekers are non-students” would have exactly the same truth value. These two sentences are logically equivalent and hence the inference from the truth of one to the truth of the other, or from the falsity of one to the falsity of the other, is valid.

As well, the inference from an O sentence to its contrapositive is also valid. We can see this clearly from the following example: from the assumption that “Some pets are not cats” is true (and as a matter of fact it is true) surely it follows that “Some non-cats are not non-pets” must also be true. We have here a valid inference. Again we leave the quality and the quantity of the derived position the same as they were in the original.

The inference from the I sentence to its contrapositive, however, is not valid. This may not be obvious at first glance. It may not be clear, for example, that if “Some women are firefighters” is true, that it is not the case that its contrapositive “Some non-firefighters are non-women” must also be true. A different example may help. It should be clear that if “Some human beings are non-professional firefighters” it does not follow that “Some professional fire-fighters are non-human beings” must also be true; and as a matter of fact it is false.

In order to make it clear that the immediate inference from an I sentence to its contrapositive is not valid, we resorted to using a sentence with a P-term that is itself a complement term, that is, a term with a “non” prefix. Recall that two “nons” cancel each other out. So following the steps in deriving the contrapositive, we first replaced the P-term with its complement. But instead of replacing the P-term, namely, “non-professional firefighters” with its complement, “non-non-professional firefighters” we simply replaced it with “professional firefighters.” Then we replaced the subject term, “human beings” with its complement, “non-human beings” and then we switched the terms around.

One interesting moral of this is that S and P class terms may be complement class terms. This means that if we begin with a categorical sentence whose terms are complement classes, we can still make valid inferences from them. For example, the contraposition of the A sentence “All non-S are non-P” is the sentence, “All P is S.”

Now what about the immediate inference from an E sentence to its contrapositive? Is this a valid move? It is not, without an appropriate qualification, or more precisely without what we called above a “limitation.” Clearly, “No professional firefighters are non-human beings” is true, but does it follow that “No humans are non-professional firefighters” must also be true, since it is in fact false. So the immediate inference of contraposition does not hold for the E sentence. But if we change the quantity of the derived contrapositive from universal to existential, from “No” to “Some,” then the immediate inference is valid, but valid by limitation. If the sentence “No professional firefighters are non-humans,” were true, then surely it follows validly that “Some humans are not professional firefighters” is true; and if the first were false the second would be as well.

In summary we have learned the following about the immediate inferences of conversion, obversion, and contraposition:

1. **Conversion:** Valid for the E and I sentences, not valid for the O sentence, and valid for the A sentence only by limitation. Since the E and the I sentences are logically equivalent to their respective converses, either sentence can be substituted for its converse, and vice versa. The A sentence is not logically equivalent to its limited converse.

2. **Obversion:** Valid for the A, E, I, and O sentences. Since all of these sentences are logically equivalent to their respective obverses, each of them can be substituted for its obverse, and vice versa.

3. **Contraposition:** Valid for the A and O sentences, not valid for the I sentence, and valid for the E sentence only by limitation. Since the A and the O sentences are logically equivalent to their respective contrapositives, either sentence can be substituted for its contrapositive, and vice versa. The E sentence is not logically equivalent to its limited contrapositive.

| | |
|------------------------|----------------------------------|
| Conversion: | |
| A: All S is P: | Some P are S (limit) |
| E: No S is P: | No P are S |
| I: Some S is P: | Some P are S |
| O: Some S is not P: | (not valid) |
| Obversion: | |
| A: All S is P: | No S are non-P |
| E: No S is P: | All S are non-P |
| I: Some S is P: | Some S are not non-P |
| O: Some S is not P: | Some S are non-P |
| Contraposition: | |
| A: All S is P: | All non-P are non-S |
| E: No S is P: | Some non-P are not non-S (limit) |
| I: Some S is P: | (not valid) |
| O: Some S is not P: | Some non-P are not non-S |

| | A | E | I | O |
|----------------|---|---|---|---|
| Conversion | I | V | V | I |
| Obversion | V | V | V | V |
| Contraposition | V | I | I | V |

To repeat: if we do not make the existential assumption, the immediate inferences of conversion, obversion and contraposition are less affected than in the square of opposition. In the latter, only contradiction survives when the existential assumption is not made. In the case of the other immediate inferences, there are only two casualties, the conversion of an A sentence and the contraposition of an E Sentence, both of which require going from a universal to an existential quantifier to be valid. And if the existential assumption is not made, neither of these moves from the universal to the existential quantifier is valid. In both cases we are moving from a sentence that does not assert that its subject class has members, to a sentence that does make this sentence. Consider this example: the converse (by limitation) of “All Martians are bald” is “Some bald people are Martians.” Certainly the first of these sentences is true because its subject class is empty and the second is false as a matter of fact. Just as clearly, the

contraposition of the E sentence “No Martians are bald” is true, since its subject class is empty, while its corresponding contrapositive (by limitation), “Some non-bald people are not non-Martians” is false.

In summary:

- **If the existential assumption is not made: only the following immediate inferences remain valid: contradictions, obversions, and valid conversions and contrapositions that do not involve limitation. We will say these inferences are valid without qualification.**
- **If the existential assumption is made (Aristotle made it), then the immediate inferences that do involve limitation are valid, and the inferences of the square of opposition other than contradiction (which is valid without qualification) are also valid. We will say that these inferences are traditionally valid.**
- **Some immediate inferences are also invalid without qualification, namely, conversions of O sentences and contrapositions of I sentences.**

Well this is all quite a lot. So let's see if we can put what we have learned into practice. It's time for the *Workbook*

STUDY GUIDE

Chapter 5 – Categorical Sentences

Standard Form Categorical Sentences

- E Sentence: “No S is P” Universal Quantity; Negative Quality
- A Sentence: “All S is P” Universal Quantity; Affirmative Quality
- I Sentence: “Some S is P” Existential Quantity; Affirmative Quality
- O Sentence: “Some S is not P” Existential Quantity; Negative Quality

Distribution Table

| |
|---|
| <p>A: S-term distributed; P-term undistributed (DU) E: both terms distributed (DD) I: both terms undistributed (UU) O: S-term undistributed; P-term distributed (UD)</p> |
|---|

Immediate Inferences

| |
|--|
| <p style="text-align: center;">Conversion: A: All S is P: Some P are S (limit) E: No S is P: No P are S I: Some S is P: Some P are S O: Some S is not P (not valid)</p> <p style="text-align: center;">Obversion: A: All S is P: No S are non-P E: No S is P: All S are non-P I: Some S is P: Some S are not non-P O: Some S is not P: Some S are non-P</p> <p style="text-align: center;">Contraposition: A: All S is P: All non-P are non-S E: No S is P: Some non-P are not non-S (limit) I: Some S is P: (not valid) O: Some S is not P: Some non-P are not non-S</p> |
|--|

| | A | E | I | O |
|----------------|---|---|---|---|
| Conversion | I | V | V | I |
| Obversion | V | V | V | V |
| Contraposition | V | I | I | V |